ECE 145b Homework #2 Solutions

#1

$\sigma_x = 2mV$ Error if $+10mV$ received and $V_{noise} \le -10mV$
or
$-10mV$ received and $V_{noise} \ge 10mV$

$$P[x \ge 10mV] = P[x \le -10mV] = 1 - \Phi\left(\frac{x-\mu}{\sigma_x}\right) = 1 - \Phi\left(\frac{10-0}{2}\right) = Q(5)$$

from statistics: $Q(5) \approx 2.86 \times 10^{-7}$

for either $\pm 10mV$,

$$\underline{P_{error} = 2.86 \times 10^{-7}}$$

#2

$$\tilde{S}_{VinVin} = 4kTR_0FG$$

$R_0 = 50\Omega$, $F=2$, $G=10^5$, assume $T=300K$

$$\underline{\tilde{S}_{VinVin} = 1.66 \times 10^{-13}\ V^2/Hz}$$

a) $V_{out_1}(jf) = h_1(j\omega)V_{in}(j\omega)$

$$\tilde{S}_{Vout1Vout1} = |h_1(j\omega)|^2 \tilde{S}_{VinVin}$$

$h(j\omega)$ should be $\frac{1}{1+j\frac{f}{f_0}}$ or $\frac{1}{1+j\frac{2\pi f}{2\pi f_0}}$
so 3 dB BW is @ f_0

$$\tilde{S}_{Vout1Vout1} = h_1(j\omega)h_1^*(j\omega)\,\tilde{S}_{VinVin}$$

$$= \left(\frac{1}{1+j\frac{f}{f_1}}\right)\cdot\left(\frac{1}{1-j\frac{f}{f_1}}\right)\tilde{S}_{VinVin}$$

$$\underline{\tilde{S}_{Vout1Vout1} = \frac{\tilde{S}_{VinVin}}{1+\left(\frac{f}{f_1}\right)^2}}$$ similarly, $$\underline{\tilde{S}_{Vout2Vout2} = \frac{\tilde{S}_{VinVin}}{1+\left(\frac{f}{f_2}\right)^2}}$$

$$\tilde{S}_{Vout1Vout2} = V_{out1}(j\omega)V_{out2}^*(j\omega) = h_1(j\omega)V_{in}(j\omega)h_2^*(j\omega)V_{in}^*(j\omega)$$

$$\underline{\tilde{S}_{Vout1Vout2} = \frac{\tilde{S}_{VinVin}}{(1+j\frac{f}{f_1})(1-j\frac{f}{f_2})}}$$

b) $\tilde{S}_{Vout3Vout3} = (V_{out1}+V_{out2})(V_{out1}^*+V_{out2}^*)$
$= \tilde{S}_{Vout1Vout1} + \tilde{S}_{Vout2Vout2} + 2Re[\tilde{S}_{Vout1Vout2}]$

$$\tilde{S}_{Vout3Vout3} = \left[\frac{1}{1+f/f_1} + \frac{1}{1+f/f_2} + 2\frac{1+\frac{f}{f_1}\frac{f}{f_2}}{(1+f/f_1)(1+f/f_2)^2}\right] \times \tilde{S}_{VinVin}$$

#3

$$S_{VV_{noise}} = 4KTR \qquad R_{gen} = 100\Omega$$

$$S_{VV_{noise}} = 1.66 \times 10^{-18}\ V^2/Hz = 1.66 \times 10^{-18}\ V^2 \text{ in 1 Hz BW}$$

$$\frac{S}{N} = \frac{|V_{gen}|^2 (V^2)}{|V_{noise}|^2 (V^2)} = \frac{(1V)^2}{(1.66 \times 10^{-18}\ V^2)} = 6.04 \times 10^{17} = 177.8\,dB$$

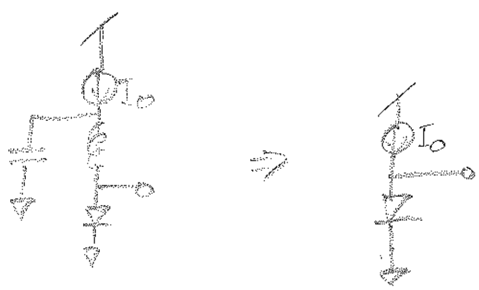
$V_{noise,rms} = \sqrt{S_{VV} \Delta f}$, signal should be left in RMS as well

#4

$$\tilde{S}_{vinvin} = 4KTR_0 F G = 1.66 \times 10^{-13}\ V^2/Hz \quad @\ T = 300K$$

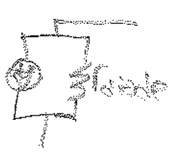
$$\tilde{S}_{voutvout} = ||h(jf)||^2 \tilde{S}_{vinvin} = \frac{1.66 \times 10^{-13}\ V^2/Hz}{1 + \left(\frac{f}{1GHz}\right)^2}$$

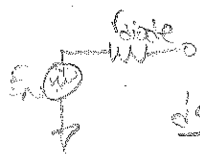
#5



$$V_{diode} = \frac{KT}{qI_0}$$

$$\frac{d\langle I_N I_N^*\rangle}{df} = 2qI$$





$$\frac{d\langle E_N E_N^*\rangle}{df} = \frac{d\langle I_N I_N^*\rangle}{df} \cdot r_{diode}^2 = \frac{2K^2T^2}{qI_0}$$

$$\frac{dP_{diode}}{df} = \frac{d\langle E_N E_N^*\rangle}{4R_{diode}} = \frac{KT}{2}\ [J]$$

Noise is ½ what's expected for resistor of same impedance.
Due to 1-way current flow

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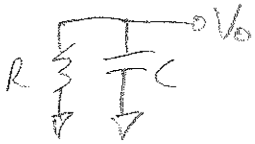

$$Z = R \,\|\, \frac{1}{sC} = \frac{R}{1+sRC} = \frac{R}{1+j2\pi fRC}$$

$$Re(Z) = \frac{R}{1+4\pi^2 f^2 R^2 C^2}$$

$$\frac{d\langle E_N, E_N^*\rangle}{df} = 4KT\,Re(Z) = \frac{4KTR}{1+4\pi^2 f^2 R^2 C^2} \quad \left(\frac{V^2}{Hz}\right)$$

$$|E_N|^2 = \int_0^\infty \frac{4KTR}{1+4\pi^2 f^2 R^2 C^2}\, df$$

$$|E_N|^2 = \frac{4KTR\pi}{2\sqrt{4\pi^2 R^2 C^2}} = \frac{KT}{C}$$

$$\underline{V_N = \sqrt{\frac{KT}{C}}}$$

or from notes:

$$\langle E \rangle = \frac{KT}{2} \qquad \langle E \rangle = \frac{1}{2} C V^2$$

$$\frac{KT}{2} = \frac{CV^2}{2} \qquad V = \sqrt{\frac{KT}{C}}$$