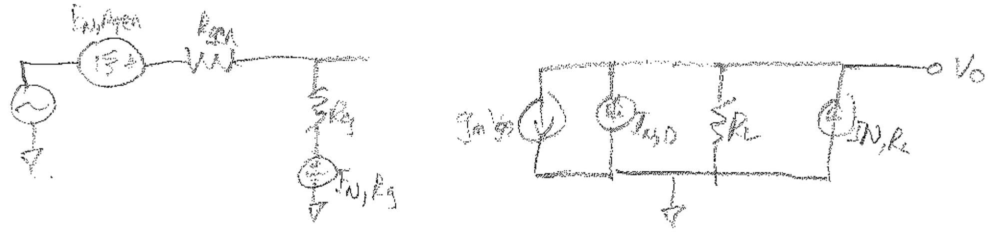
ECE 145b HW#3 Solutions

#1



$$\tilde{S}_{out,Rgen} = 4KTR_{gen}\|A_v\|^2$$

$$\tilde{S}_{out,R_L} = \frac{4KT}{R_L}(R_L)^2 \qquad A_v = \left(\frac{R_g}{R_g+R_{gen}}\right)(-g_m R_L)$$

$$\tilde{S}_{out,D} = I_{N,D}\cdot R_L^2 = 4KT\Gamma g_m R_L^2$$

$$\tilde{S}_{out,R_g} = 4KTR_g\left(\frac{R_{gen}}{R_g+R_{gen}}\right)^2(g_m R_L)^2$$

$$\tilde{S}_{ampout} = \tilde{S}_{out,R_L} + \tilde{S}_{out,D} + \tilde{S}_{out,R_g}$$

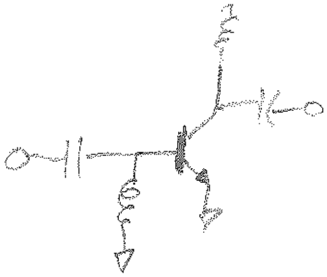
$$\tilde{S}_{amp,in} = \frac{\tilde{S}_{ampout}}{\|A_v\|^2} = 4KTR_g\left(\frac{R_{gen}}{R_g}\right)^2 + \left(\frac{R_g+R_{gen}}{R_g}\right)^2\left(\frac{1}{g_m R_L}\right)^2\left[4KT\Gamma g_m R_L^2 + 4KTR_L\right]$$

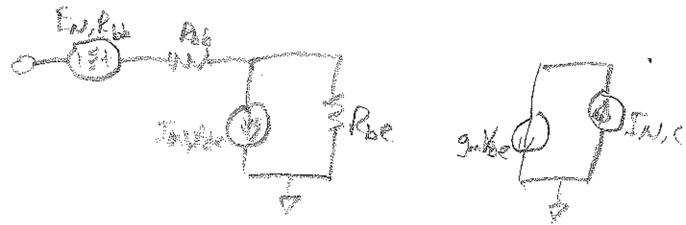
$$\tilde{S}_{amp,in} = 2.475\times10^{-18}\ V^2/Hz$$

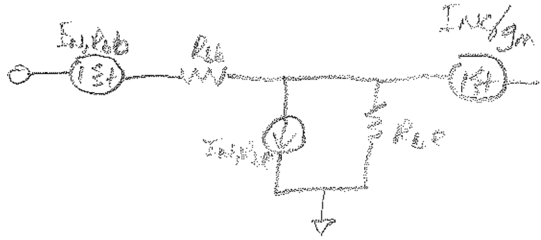
$$\tilde{S}_{total,in} = \tilde{S}_{amp,in} + \underset{(4KTR_{gen})}{\tilde{S}_{Rgen}} = 1.24\times10^{-17}\ V^2/Hz$$

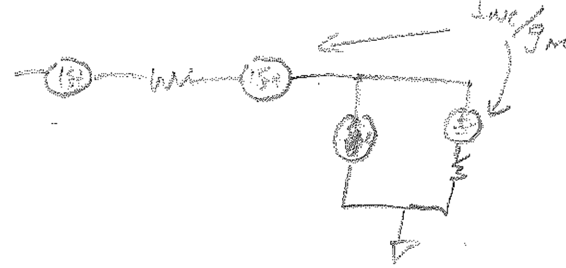
$$SNR = \frac{\tilde{S}_{sig}}{\tilde{S}_{totalnoise,in}} = \frac{(1mV)^2}{(1.24\times10^{-17}V^2/Hz)(4000-400\ Hz)} = 73.5dB$$

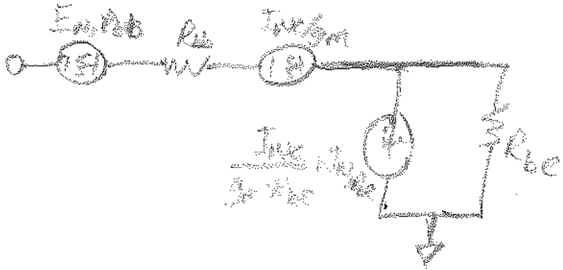
#2

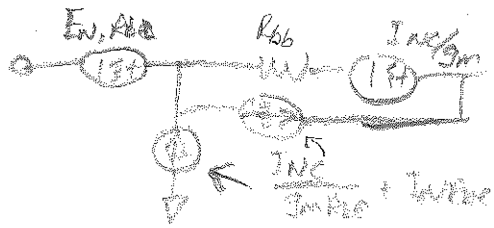


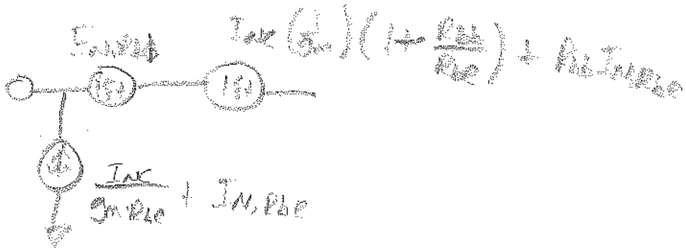




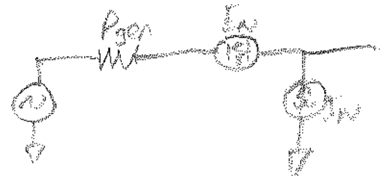








$$\tilde{S}_{EN_{in}} = 4KTR_{bb} + \frac{2KTg_m}{g_m^2}\left(1 + \frac{R_{bb}^2}{R_{be}^2}\right) + \frac{2KT R_{bb}^2}{R_{be}}$$



$$\tilde{S}_{EN} + \tilde{S}_{IN}\|R_g\|^2 + 2\,\mathrm{Re}(\tilde{S}_{ENIN} R_g^*) = \tilde{S}_{EN,total} = 4KT(R_{bb}) + \frac{2KT}{g_m}\left(1 + \frac{R_{bb}^2}{R_{be}^2}\right) + \frac{2KT R_{bb}^2}{R_{be}} +$$

$$\left(\frac{2KTg_m}{g_m^2 R_{be}^2} + \frac{2KT}{R_{be}}\right)R_{gen}^2 + 2\left(\frac{2KTg_m(1 + \frac{R_{bb}}{R_{be}})}{g_m^2 R_{be}} + \frac{2KT R_{bb}}{R_{be}}\right)R_{gen}$$

$$\tilde{S}_{EN} = 4KTR_{bb} + \frac{2KT}{g_m}\left(1 + \frac{R_{bb}^2}{R_{be}^2}\right) + \frac{2KT R_{bb}^2}{R_{be}} = \underline{1.9 \times 10^{-18}\ V^2/Hz}$$

$$\tilde{S}_{IN} = \frac{2KT}{g_m R_{be}^2} + \frac{2KT}{R_{be}} = \underline{3.22 \times 10^{-24}\ A^2/Hz}$$

$$\tilde{S}_{ENIN} = \frac{2KT(1 + R_{bb}/R_{be})}{g_m R_{be}} + \frac{2KT R_{bb}}{R_{be}} = \underline{4.04 \times 10^{-22}\ J}$$

$$\tilde{S}_{EN,total} = 3.11 \times 10^{-18}\ V^2/Hz$$

$$V_{N} = 1.76\ nV/\sqrt{Hz}$$

#2

c) $F = \dfrac{\bar{S}_{total\ noise}}{\bar{S}_{v_{gen,in}}} = \dfrac{\bar{S}_{en,total} + 4kT(500)}{4kT(500)} = 1.375$

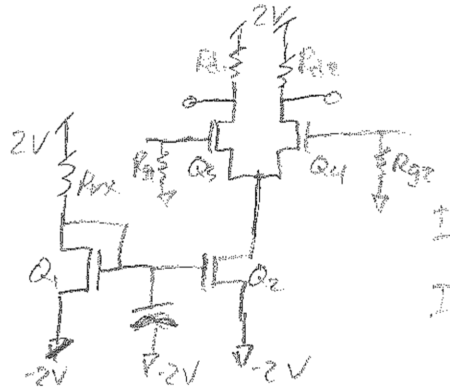
d) $F_{min} = 1 + \dfrac{1}{4kT}\left[2\sqrt{\bar{S}_{ENEN}\,\bar{S}_{ININ} - \cancelto{0}{\mathrm{Im}\{\bar{S}_{ENIN}\}^2}} + 2\,\mathrm{Re}\{\bar{S}_{ENIN}\}\right]$

$F_{min} = 1.3475$

$Z_{opt} = \sqrt{\dfrac{\bar{S}_{ENEN}}{\bar{S}_{ININ}} - \cancelto{0}{\mathrm{Im}\left\{\dfrac{\bar{S}_{ENIN}}{\bar{S}_{ININ}}\right\}^2}} - j\,\cancelto{0}{\dfrac{\mathrm{Im}[\bar{S}_{ENIN}]}{\bar{S}_{ININ}}}$

$Z_{opt} = 768.2\ \Omega$

#3



$I_{Q_3} = I_{Q_4} = 100\mu A$

$I_{Q_1} = I_{Q_2} = 200\mu A$

a)

$$200\mu A = (1mA/V^2)(V_{gs_{1,2}} - V_{th})^2$$

$$V_{gs_{1,2}} = \sqrt{.2} + 0.3V = 0.747V$$

$$R_{xx} = \frac{4V - 0.747V}{200\mu A} = 16.265\,k\Omega$$

$$100\mu A = (1mA/V^2)(V_{gs_{3,4}} - V_{th})^2$$

$$V_{gs_{3,4}} = \sqrt{.1} + 0.3V = 0.616V$$

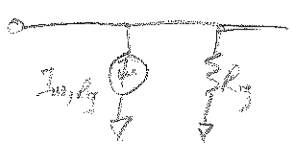
$$R_{d1} = R_{d2} = \frac{0.5V}{100\mu A} = 5k\Omega$$

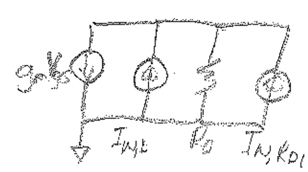
$$g_{m3} = g_{m4} = \frac{2 I_D}{V_{gs} - V_{th}} = 633\mu S$$

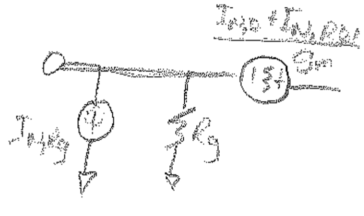
b) for Q_2 $R_{ds} = \infty$, $A_{v,diff} = -g_{m3} R_{d1} = -g_{m4} R_{ds} = -3.16$

c) any noise from current bias Q_1/Q_2 will be common mode to V+ & V−, can be neglected

for V+







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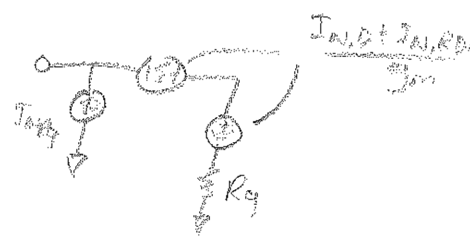
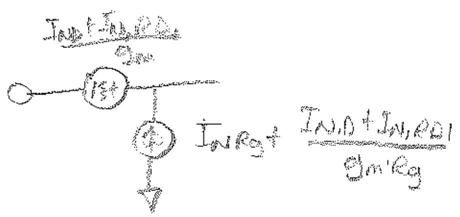
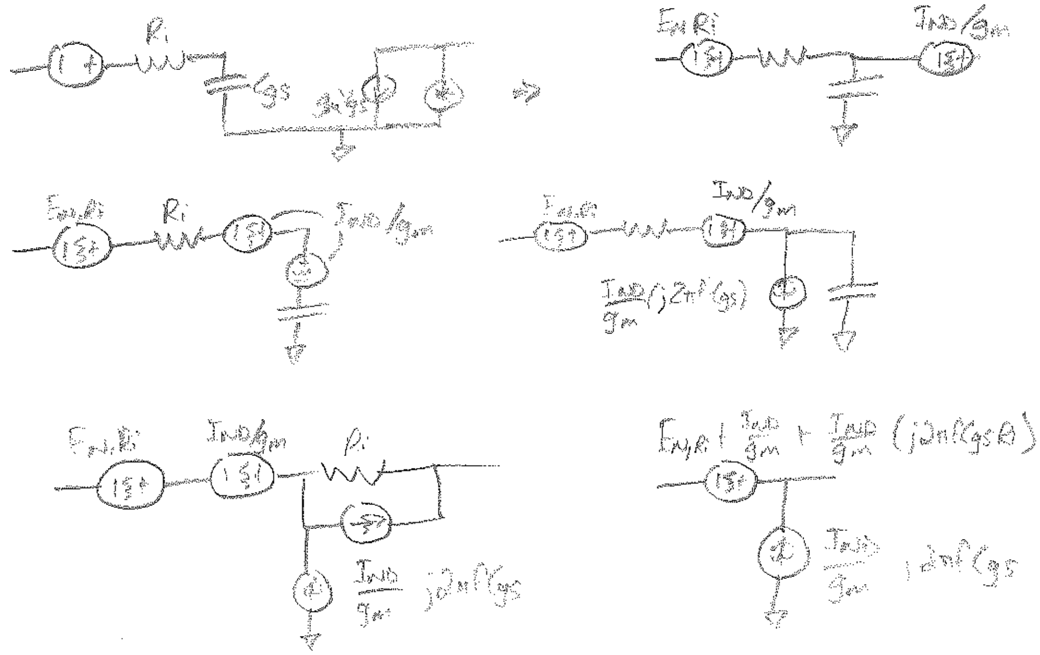




HW#3 Extra Problems

#1

a)



$$\tilde{S}_{EN,EN} = 4KTR_i + \frac{4KT\Gamma}{g_m}\left(1 + 4\pi^2 f^2 C_{gs}^2 R_i^2\right)$$

$$\tilde{S}_{IN,IN} = \frac{4KT\Gamma}{g_m}\left(4\pi^2 f^2 C_{gs}^2\right)$$

$$\tilde{S}_{EN,IN} = \frac{4KT\Gamma}{g_m}\left(1 + j2\pi f C_{gs} R_i\right)\left(j2\pi f C_{gs}\right)^* = \frac{4kT\Gamma}{g_m}\left(4\pi^2 f^2 C_{gs}^2 R_i - j2\pi f C_{gs}\right)$$

b)

$f_T = 100GHz$ $\quad 2\pi f_T = \frac{g_m}{C_{gs}}$ $\quad g_m = 100mS$, $C_{gs} = 159fF$, $R_i = 10\Omega$, $\Gamma = 1$ $f = 10GHz$

$$\tilde{S}_{EN,total} = \tilde{S}_{EN} + \tilde{S}_{IN} R_{gen}^2 + 2\cdot Re\{\tilde{S}_{EN,IN} R_{gen}\}$$

$$= 4KT\left[R_i + \frac{\Gamma}{g_m}\left(1 + 4\pi^2 f^2 C_{gs}^2 R_i^2\right) + \frac{\Gamma}{g_m}\left(4\pi^2 f^2 C_{gs}^2\right)R_{gen}^2 + 2\left(\frac{\Gamma}{g_m}\cdot 4\pi^2 f^2 C_{gs}^2 R_i\right)\cdot R_{gen}\right]$$

$$\tilde{S}_{EN,total} = 3.91\times10^{-19}\ V^2/Hz$$

$$F = \frac{\tilde{S}_{EN,total} + \tilde{S}_{gen,in}}{\tilde{S}_{gen,in}} = 1.472$$

#1

c) $F_{min} = 1 + \frac{1}{qkT}\left[2\sqrt{\tilde{S}_{EN}\tilde{S}_{IN} - \mathrm{Im}\{\tilde{S}_{ENIN}\}^2} + 2\,\mathrm{Re}\{\tilde{S}_{ENIN}\}\right]$

$\hat{S}_{EN}$ 3.33×10^{-19} V^2/Hz

$\tilde{S}_{IN} = 1.65\times10^{-23}$ A^2/Hz

$\mathrm{Re}\{\tilde{S}_{ENIN}\} = 1.65\times10^{-22}$ W/Hz

$\mathrm{Im}\{\tilde{S}_{ENIN}\} = -1.65\times10^{-21}$ W/Hz

$$\underline{F_{min} = 1.221}$$

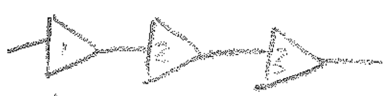
$$Z_{opt} = \sqrt{\frac{\tilde{S}_{EN}}{\tilde{S}_{IN}} - \left(\frac{\mathrm{Im}\{\tilde{S}_{ENIN}\}}{\tilde{S}_{IN}}\right)^2} - j\,\frac{\mathrm{Im}\{\tilde{S}_{ENIN}\}}{\tilde{S}_{IN}}$$

$$\underline{Z_{opt} = 100.6 - j100.1\,\Omega}$$

d) $$\tilde{S}_{IN,total} = \tilde{S}_{IN} + \frac{\tilde{S}_{EN}}{Z_g^2} + 2\,\mathrm{Re}\left\{\frac{\tilde{S}_{ENIN}}{Z_g}\right\}$$

$$\underline{\tilde{S}_{IN,total} = 1.65\times10^{-27}\ A^2/Hz}$$

#2



G 5 10 15 dB

F 3 6 9 linear

5dB → 3.16

10dB → 10

$$F_{overall} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

$$\underline{F_{overall} = 4.84}$$

3

$I_c = 1mA$
$R_{bb} = 100\Omega$
$B = 100$

$g_m = \frac{I_c}{26mV} = 38.5mS$
$R_{be} = \frac{B}{g_m} =$

a)

from #2 b

$F_{min} = 1.3475 \qquad Z_{opt} = 768.2\Omega$

b) $\tilde{S}_{EN,total} = \tilde{S}_{EN} + \tilde{S}_{IN} \cdot (1k\Omega)^2 + 2 \cdot Re\{S_{EN IN} \cdot (1k\Omega)\}$

$\tilde{S}^2_{EN,total} = 5.93 \times 10^{-18} \, V^2/Hz$

c) $F = \frac{\tilde{S}_{EN,total} + \tilde{S}_{Rgen}}{\tilde{S}_{Rgen}}$

$g_m = \frac{I_c}{V_T}$

$R_{be} = \frac{B}{g_m} = \frac{B \cdot V_T}{I_c}$

$\frac{dF}{dI_c} = \frac{d\tilde{S}_{EN,total}}{dI_c} = 0$

$$\tilde{S}_{EN,total} = 4KTR_{bb} + \frac{2KTV_T}{I_c}\left(1 + \frac{R_{bb}^2}{\left(\frac{BV_T}{I_c}\right)^2}\right) + \frac{2KTR_{bb}^2}{\left(\frac{BV_T}{I_c}\right)} + \left(\frac{2KT}{\frac{I_c}{V_T}\cdot\left(\frac{BV_T}{I_c}\right)^2} + \frac{2KT}{\left(\frac{BV_T}{I_c}\right)}\right)R_{gen}^2$$
$$+ 2\left(\frac{2KT\left(1 + \frac{R_{bb}}{\left(BV_T/I_c\right)}\right)}{\frac{I_c}{V_T}\,\frac{BV_T}{I_c}} + \frac{2KTR_{bb}}{\left(\frac{BV_T}{I_c}\right)}\right)R_{gen}$$

$$\frac{\partial \tilde{S}_{EN,total}}{\partial I_c} = \frac{-2KTV_T}{I_c^2} + \frac{2KV_TR_{bb}^2}{(BV_T)^2} + \frac{2KTR_{bb}^2}{BV_T} + \left(\frac{2KT}{B^2V_T} + \frac{2KT}{BV_T}\right)R_{gen}^2$$
$$+ 2\left(\frac{2KTR_{bb}}{B^2V_T} + \frac{2KTR_{bb}}{BV_T}\right)R_{gen} = 0$$

$\underline{I_c = 235.2\mu A}$