ECE 2C Problem Set #2 Solution

#1

For Mobility Limited FET

$$I_d = (\mu C_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$$

$$\frac{\mu C_{ox} W_g}{2L_g} = 1mA/V^2, \quad V_{th} = 0.3V, \quad \lambda = 0.1V^{-1} \leftarrow \text{Given!}$$

(a)

$V_{ds} = 2V$ & $I_d = 0.5mA$

$\uparrow$ DC bias $\qquad \uparrow$ Drain Current!

$$\Rightarrow 0.5mA = 1mA/V^2 \times (V_{gs} - 0.3V)^2 (1 + 0.1V^{-1} \cdot 2V) \leftarrow I_d$$

$$\Rightarrow V_{gs} = \sqrt{\frac{0.5}{1.2}} + 0.3 = \underline{0.945V}$$

Transconductance

$$\Rightarrow g_m = \frac{\partial I_D}{\partial V_{GS}} = (\mu C_{ox} W_g / L_g)(V_{gs} - V_{th})(1 + \lambda V_{DS})$$

$$= 2mA/V^2 \cdot (0.945 - 0.3)(1 + 0.2) = \underline{1.548 mA/V}$$

Output Conductance

$$G_{ds} = \frac{\partial I_D}{\partial V_{DS}} = \frac{\lambda I_D}{1 + \lambda V_{DS}} = \frac{0.1V^{-1} \times 0.5mA}{1 + 0.2} = \underline{0.416 mA/V}$$

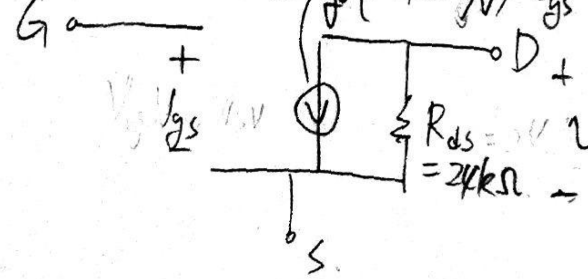
$$= \frac{1}{R_{ds}} \Rightarrow \underline{R_{ds} = 2.4k\Omega}$$

$\uparrow$

$V/mA = 10^3 V/A$

(b) <Small Signal Circuit>

$g_m V_{gs} = (1.548 mA/V) \cdot v_{gs}$



$$v_{out} = -g_m v_{gs} R_{ds}$$

#2.

For Velocity limited FET

$I_d = v_{sat} C_{ox} W_g (V_{gs} - V_{th})(1 + \lambda V_{ds})$ ← Current equation

$v_{sat} C_{ox} W_g = 0.5\,mA/V$, $\lambda = 0.1\,V^{-1}$, $V_{th} = 0.3V$ ← given

(a)

$V_{ds} = 1V$, $V_{gs} = 0.5\,V$

Transconductance

$\Rightarrow g_m = \frac{\partial I_D}{\partial V_{GS}} = \underbrace{C_{ox} v_{sat} W_g}_{0.5mA/V} (1 + \underbrace{\lambda}_{0.1V^{-1}} \underbrace{V_{DS}}_{1V}) = \underline{0.55\,mA/V}$

to get G_{ds}, we need I_d value first,

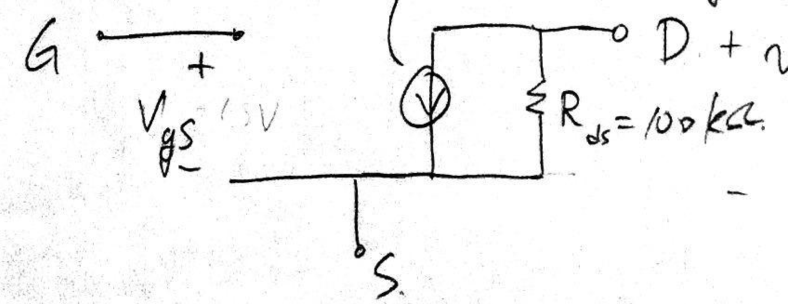
$I_d = 0.5mA/V \cdot (0.5V - 0.3V)(1 + 0.1) = \underline{0.11mA}$.

Output Conductance

$\Rightarrow G_{ds} = \frac{\lambda I_D}{1 + \lambda V_{DS}} \left(= \frac{1}{R_{ds}} \right) = \frac{0.1V^{-1} \cdot 0.11mA}{1.1} = \underline{0.01\,mA/V}$

(b)

$g_m V_{gs} = 0.55mA/V \cdot V_{gs}$. $R_{ds} = 100\,k\Omega$.

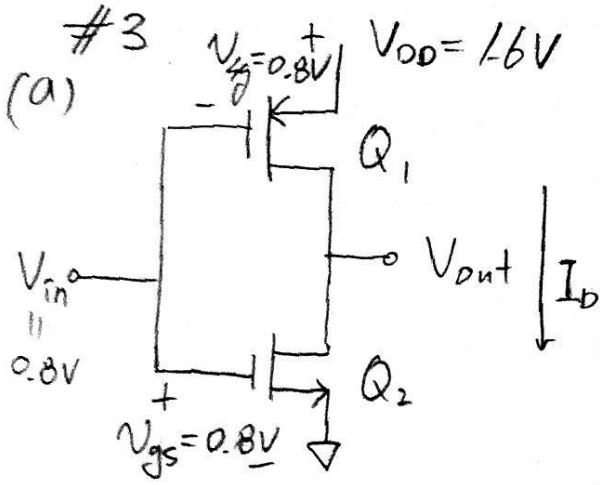


$+ \quad v_{out} = -g_m V_{gs} R_{ds}$

$- \quad = -(0.55mA) \cdot (100k\Omega) V_{gs}$

#3

(a)



Given Values

L_g	45nm	45nm
t_{ox}	0.8nm	0.8nm
λ	$0.3V^{-1}$	$0.3V^{-1}$
v_{sat}	10^7 cm/s	0.5×10^7 cm/s
W_g	5 μm	10 μm
V_{th}	0.4 V	-0.4 V

(b) For DC biasing,

$$I_{D,NFET} = I_{D,PFET}$$

$$\Rightarrow C_{ox} v_{sat} W_g (1+\lambda V_{DS})(V_{gs} - V_{th})$$

same $\quad v_{sat,N} = 2 v_{sat,p} \quad W_{g,N} = \frac{1}{2} W_{g,P}$

Thus,

$$C_{ox} v_{sat,N} W_{g,N} (1+\lambda v_{DS,N})(0.8-0.3) = C_{ox} \cdot \underbrace{\tfrac{1}{2} v_{sat,N}}_{v_{sat,P}} \cdot \underbrace{2W_{g,N}}_{W_{g,P}} (1+\lambda V_{SD,P})(0.8-0.3)$$

→ All cancelled out ⇒ $V_{DS,NFET} = V_{SD,PFET} = 0.8V$

Because $C_{ox} v_{sat} W_g$ products are same for both Q_1 and Q_2 with same V_{gs}, they have same as 0.8V.

$$\boxed{\therefore V_{out} = V_{DS} = 0.8V}$$ ← DC Value of V_{out}.

To obtain drain current I_D, we need to calculate Cox value.

$$\Rightarrow C_{ox} = \frac{\epsilon_0 \epsilon_r}{t_{ox}} = \frac{3.8 \times 8.85 \times 10^{-12} F/m}{0.8nm} = 0.042 F/m^2 = 42 fF/\mu m^2$$

($\epsilon_r = 3.8$)

$\Rightarrow$ $C_{ox} V_{sat} W_g$ products for PFET & NFET are same!

$$C_{ox} V_{sat} W_g = \left(\frac{42\,fF}{\mu m}\right)\left(10^7 \frac{cm}{s}\right)\left(\frac{5\,\mu m}{1\,\mu m}\right) = \left(\frac{42\times10^{-15}}{10^{-6}}\right)\left(10^7\cdot10^{-2}\frac{m}{s}\right)\cdot 5$$

$$= \underline{21\,mA/V}$$

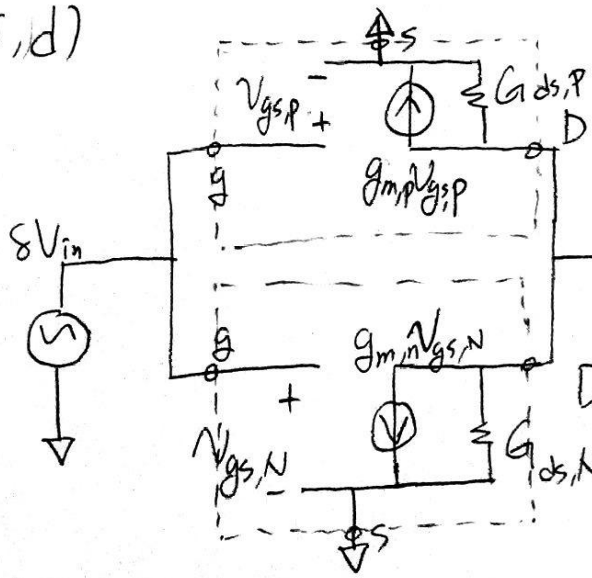
$\Rightarrow$ Drain Current

$$I_{D,NFET} = I_{D,PFET} = C_{ox} V_{sat} W_g \left(1+\lambda V_{DS,N}\right)\left(V_{gs} - V_{th}\right)$$

$$= 21\,mA/V \cdot \left(1 + 0.3V^{-1}\cdot 0.8V\right)\left(0.8V - 0.3V\right)$$

$$\therefore \boxed{I_D = 13.02\,mA}$$

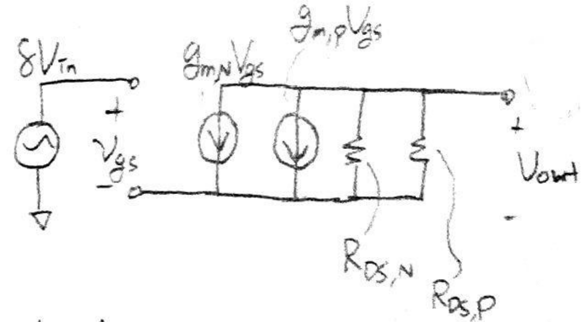
(c, d)



$g_m V_{gs} = 26.04\,mA/V \cdot V_{gs}$

$R_{ds} = \frac{1}{G_{ds}} = 317\,\Omega$

$\Rightarrow$



$$g_{m,N} = \frac{\partial I_D}{\partial \underset{\substack{\uparrow \\ DC\ V_{GS}}}{V_{GS}}} = C_{ox} V_{sat} W_g \left(1+\lambda V_{DS}\right) = 21\frac{mA}{V}\cdot\left(1+0.3\cdot0.8\right) = \boxed{26.04\,mA/V}$$

$$R_{ds,N} = \frac{1}{G_{ds}} = \frac{1}{\left(\frac{\partial I_D}{\partial V_{DS}}\right)} = \frac{1}{\left(\frac{\lambda I_D}{1+\lambda V_{DS}}\right)} = \frac{\cancel{\left(1+\lambda V_{DS}\right)}}{\underset{0.3V^{-1}}{\lambda}\underbrace{\left(\underset{21mA}{C_{ox}V_{sat}W_g}\cancel{\left(1+\lambda V_{DS}\right)}\underset{0.8-0.3}{\left(V_{GS}-V_{th}\right)}\right)}_{I_D}} = \boxed{317\,\Omega}$$

Q_1 & Q_2 have same characteristics.

$\therefore$ ($g_{m,N} = g_{m,P} = 26.04\,mA/V$

$R_{ds,N} = R_{ds,P} = 317\,\Omega$)

(e)

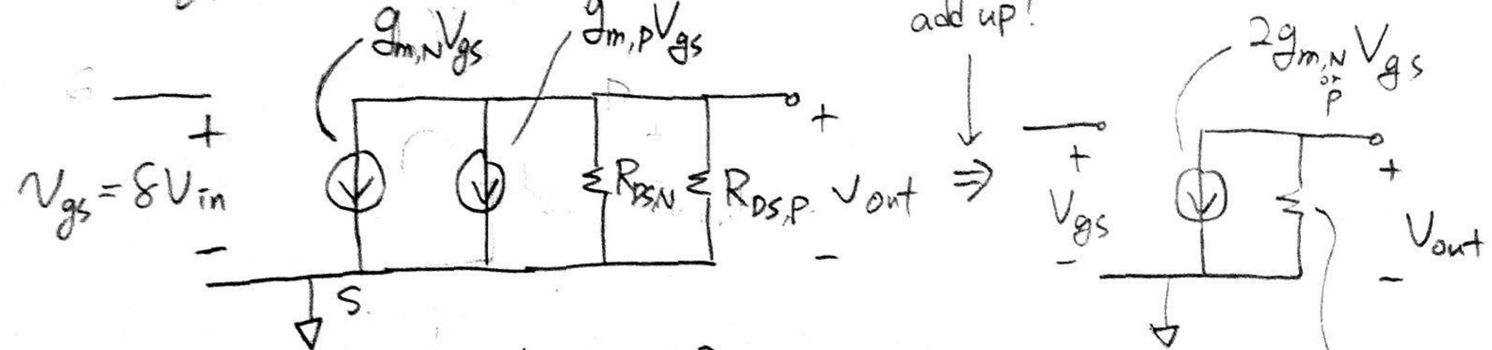
AC Voltage : δV_{in}

$\Rightarrow V_{DC+AC} = 0.8V + \delta V_{in}$

For small signal analysis,

&

For equivalent circuit,



Current across $R_{ds} = -2g_{m,N}V_{gs} = -2g_{m,P}V_{gs}$

($g_m \rightarrow$ identical

$R_{DS} = \frac{1}{G} \Rightarrow$ identical

for both N&P FET)

$\therefore$ Thus, $\boxed{V_{out} = -g_m V_{gs} R_{DS}}$

$= -26.04\,mA/V \cdot \delta V_{in} \cdot 317\,\Omega$

$= 8.25\,\delta V_{in}$

#4.

$$I_a = k_p \left(V_g + \frac{V_a}{\mu} \right)^{3/2}$$

(a)

Output Conductance $= \frac{\partial I_a}{\partial V_a} = \frac{\partial}{\partial V_a}\left[k_p \left(V_g + \frac{V_a}{\mu} \right)^{3/2} \right]$

$$G = \frac{3}{2}\frac{k_p}{\mu}\left(V_g + \frac{V_a}{\mu} \right)^{1/2}$$ //

Transconductance $= \frac{\partial I_a}{\partial V_g} = \frac{\partial}{\partial V_g}\left(k_p \left(V_g + \frac{V_a}{\mu} \right)^{3/2} \right)$

$$g_m = \frac{3}{2} \cdot k_p \left(V_g + \frac{V_a}{\mu} \right)^{1/2}$$ //

(b)

To get k_p, μ values, pick some data from graph.

By graph,

① At $V_g = 0V$, $V_a = 200V$, $I_a = 4.5mA$

$\rightarrow I_a = 4.5mA = k_p \left(\frac{200}{\mu} \right)^{3/2} \rightarrow (4.5mA)^2 = k_p^2 \left(\frac{200}{\mu} \right)^3$

② At $V_g = -2V$, $V_a = 200V$, $I_a = 0.5mA$.

$\rightarrow I_a = 0.5mA = k_p \left(-2 + \frac{200}{\mu} \right)^{3/2} \rightarrow (0.5mA)^2 = k_p^2 \left(\frac{200}{\mu} - 2 \right)^3$

$\Rightarrow \left(\frac{\cancel{4.5mA}^{\,9}}{\cancel{0.5mA}} \right)^2 = \frac{\cancel{k_p^2}}{\cancel{k_p^2}} \cdot \left(\frac{\frac{200}{\mu}}{\frac{200}{\mu} - 2} \right)^3 \Rightarrow 81 = \left(\frac{200}{200 - 2\mu} \right)^3$

$$(81)^{1/3} = \frac{200}{200-2\mu}$$

$$\rightarrow \frac{200}{(81)^{1/3}} = 200-2\mu \Rightarrow \therefore \mu = 100 - \frac{100}{(81)^{1/3}} = \underline{76.888}$$

&

$$\therefore k_p = \frac{4.5\times10^{-3}A}{\left(\frac{200}{76.888}\right)^{3/2}} = \underline{0.001115\ A/V^{3/2}}$$

Thus,

At $V_a = 200V$, $V_g = -0.5V$

Transconductance

$$g_m = \frac{3}{2}k_p\left(-0.5+\frac{200}{\mu}\right)^{1/2} = 0.00351A/V = \underline{3.51\,mA/V}$$

Output Conductance & slope of the graph

$$G = \frac{3}{2}\frac{k_p}{\mu}\left(-0.5+\frac{200}{\mu}\right)^{1/2} = 0.000046\ A/V = \underline{0.046\,mA/V}$$

#5.

(a) Using the work from #4,

given $V_{in\,DC} = V_g = -0.5V$ & $V_a = V_{AA} - I_a R_L = 232 - 10\times10^{-3} I_a$

For satisfying value, $V_a = 200V$

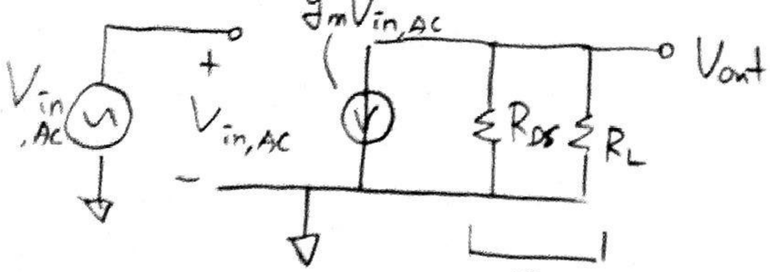
$I_a = 3.2mA$

As also mentioned, taking same tube in #4b,

$g_m = 3.51 mA/V$

$G = 0.046 mA/V \rightarrow R_{DS} = \frac{1}{G} = \underline{21.74 k\Omega}$

(b) small signal equivalent circuit



(c)

We can combine R_{DS} & $R_L \Rightarrow R_{eq} = R_{DS} // R_L$ & Let $V_{in\,AC} = \pm 1mV$.

Thus,

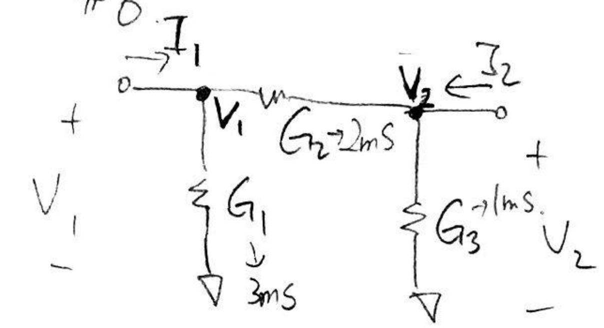
$= \left(\frac{217.4}{21.74+10}\right) k\Omega = \underline{6.85 k\Omega}$

$V_{out} = -g_m V_{in\,AC} (R_{DS} // R_L) = -3.51 \frac{mA}{V} \cdot (\pm 1mV) \cdot 6.85 k\Omega \rightarrow \frac{V}{A}$

$= \mp 24.04 mV$

$\boxed{\therefore V_{out\,AC} = -24.04 mV \cdot \cos(2\pi \cdot 1kHz \cdot t)}$

#6



$$I_1 = G_1V_1 + G_2(V_1 - V_2) = (G_1+G_2)V_1 - G_2V_2$$

$$I_2 = G_3V_2 + G_2(V_2 - V_1) = -G_2V_1 + (G_2+G_3)V_2$$

$$\Rightarrow \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} G_1+G_2 & -G_2 \\ -G_2 & G_2+G_3 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

(a)

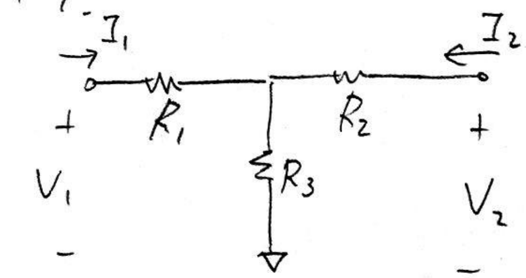
$$\Rightarrow Y = \begin{pmatrix} 5mS & -2mS \\ -2mS & 3mS \end{pmatrix} \Rightarrow \mathbf{I} = \mathbb{Y}\mathbb{V}$$

(b)

$$\mathbb{Y}^{-1}\mathbb{I} = \mathbb{V} \Rightarrow Z = \mathbb{Y}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{(15-4)\times10^{-6}}\begin{pmatrix} 3\times10^{-3}\Omega & 2\times10^{-3}\Omega \\ 2\times10^{-3}\Omega & 5\times10^{-3}\Omega \end{pmatrix}$$

$$\boxed{\therefore Z = \frac{1}{11}\begin{pmatrix} 3k\Omega & 2k\Omega \\ 2k\Omega & 5k\Omega \end{pmatrix}}$$

#7



$$\Rightarrow V_1 = I_1R_1 + (I_1+I_2)R_3 = (R_1+R_3)I_1 + R_3I_2$$

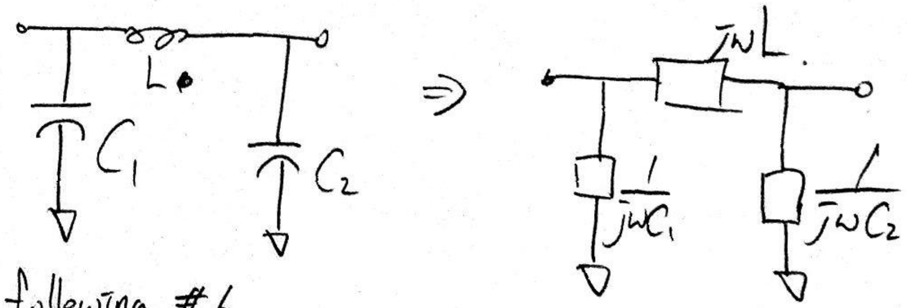
$$V_2 = I_2R_2 + (I_1+I_2)R_3 = I_1R_3 + (R_2+R_3)I_2$$

$$(b) \Rightarrow \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \underbrace{\begin{pmatrix} R_1+R_3 & R_3 \\ R_3 & R_2+R_3 \end{pmatrix}}_{Z = \begin{pmatrix} 4k\Omega & 1k\Omega \\ 1k\Omega & 3k\Omega \end{pmatrix}} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

(a)

$$\therefore \mathbb{Y} = Z^{-1} = \frac{1}{11\times10^{-6}}\begin{pmatrix} 3\times10^3 & -1\times10^3 \\ -1\times10^3 & 4\times10^3 \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 3mS & -1mS \\ -1mS & 4mS \end{pmatrix}$$

#8.



& $f = 2\,GHz$
$C_1 = 2pF$
$C_2 = 3pF$
$L = 2nH$

by following #6.

$$Y = \begin{pmatrix} \frac{1}{j\omega C_1} + j\omega L & -j\omega L \\ -j\omega L & \frac{1}{j\omega C_2} + j\omega L \end{pmatrix} = j \begin{pmatrix} \omega L - \frac{1}{\omega C_1} & -\omega L \\ -\omega L & \omega L - \frac{1}{\omega C_2} \end{pmatrix}$$

For ωL

$\rightarrow 2\pi f L = 2\pi \times 2\times 10^9 \times 2\times 10^{-9} = 8\pi = 25.13$

$$Y = j \begin{pmatrix} -14.656 & -25.13 \\ -25.13 & -1.393 \end{pmatrix}$$