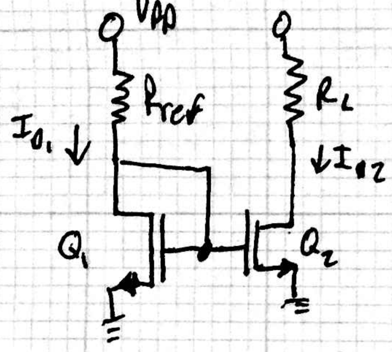
1



$$I_D = \frac{\mu C_{ox} W_G}{2L_g}(V_{gs} - V_{th})^2(1+\lambda V_{DS}),\quad V_D > V_G - V_{th}$$

$$I_D = \frac{\mu C_{ox} W_G}{2L_g}\left[2(V_{gs}-V_{th})V_{DS} - V_{DS}^2\right](1+\lambda V_{DS}) \quad V_D < V_g - V_{th}$$

$$\frac{\mu C_{ox}}{2L_g} = \frac{1\,mA}{\mu m V^2},\ \lambda = 0\,V^{-1},\ W_{g_1} = 1\,\mu m,\ V_{DD} = 3.3V,\ V_{th} = 0.30V$$

a) R_{ref} ST $I_{D_1} = 100\mu A$

$$I_{D_1} = \frac{\mu C_{ox}}{2L_g} W_{G_1}(V_{gs} - V_{th})^2(1+\cancel{\lambda}^{\,0} V_{DS})$$

$$0.1\,mA = \left(\frac{1mA}{\mu m\,V^2}\right)(1\,\mu m)(V_{gs} - 0.3V)^2 \Rightarrow V_{gs} = 0.616V$$

so $$R_{ref} = \frac{V_{DD} - V_{GS}}{I_{D_1}} = \frac{(3.3 - 0.616)V}{0.1\,mA} = \boxed{26.84\,k\Omega = R_{ref}}$$

b) assume $R_L \simeq 0$, find W_{g2} ST $I_{D_2} = 300\mu A$

$$0.3\,mA = \left(\frac{1mA}{\mu m\,V^2}\right) W_{g2}(0.616V - 0.3V)^2 \Rightarrow \boxed{W_{g2} = 3.004\,\mu m}$$

c) Constant current operation when Q point stays in saturation region.
So const. current when $V_{DS} \geq V_{DS}^{sat}$ + when $R_L \leq \frac{V_{DS}^{sat}}{I_D = 50mA}$

$$V_{DS}^{sat} = V_{gs} - V_{th} = 0.616V - 0.3V = 0.316V$$

so $$\boxed{V_D \geq 0.316V}$$

+ $$\boxed{R_L \leq 6.32\,k\Omega}$$

(2) now set $\lambda = 0.1\,V^{-1}$

a) Find R_{ref} ST $I_{D_1} = 100\,\mu A$

NB: $V_{DS_1} = V_{gs}$, $W_{g_1} = 1\,\mu m$

so $$I_{D_1} = \frac{\mu C_{ox}}{2 L_g} W_{g_1} (V_{gs} - V_{th})^2 (1 + \lambda V_{gs})$$

$$0.1\,mA = \left(\frac{1\,mA}{\mu m V^2}\right)(1\,\mu m)(V_{gs} - 0.3V)^2 (1 + (0.1 V^{-1}) V_{gs})$$

V_{gs} must be > 0 so only one sol'n remains:

$$\Rightarrow \boxed{V_{gs} = 0.607\ V}$$

b) now $$I_{D_2} = \frac{\mu C_{ox}}{2 L_g} W_{g_2} (V_{gs} - V_{th})^2 (1 + \lambda (V_{DD} - I_{D_2} R_L))$$

where we've substituted $V_{DS_2} = V_{DD} - I_{D_2} R_L$

R_L is assumed to be small, so $V_{DS_2} = V_{DD} = 3.3V$

so $$0.3\,mA = \left(\frac{1\,mA}{\mu m V^2}\right) W_{g_2} (0.607V - 0.3V)^2 (1 + \overset{[V^{-1}]}{(0.1)} \cdot (3.3V))$$

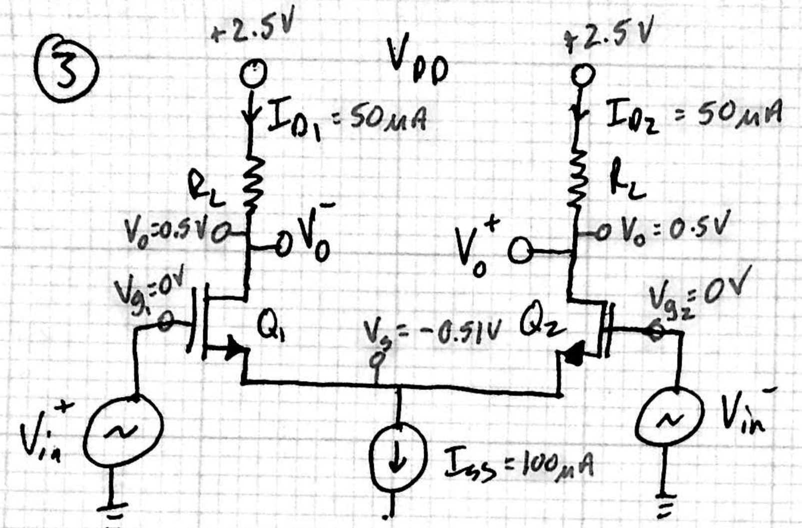
find $\boxed{W_{g_2} = 2.39\,\mu m}$

c) now let $V_{D2} = 2.3V$, $V_s = 0V$ so $V_{D2} = V_{DS_2}$

so $$I_{D_2} = \left(\frac{1\,mA}{\mu m V^2}\right)(2.39\,\mu m)(0.607V - 0.3V)^2 (1 + (0.1 V^{-1})(2.3V))$$

$$\Rightarrow \boxed{I_{D_2} = 277.1\,\mu A}$$

③



$V_{th} = 0.3V$ $\qquad \dfrac{\mu C_{ox} W_g}{2L_g} = \dfrac{1mA}{V^2}$

$\lambda = 0.1\ V^{-1}$ $\qquad V_{DD} = 2.5V$

$I_{D1} = I_{D2} = 50\mu A$

$$\text{DC}\begin{cases} V_o^+ = V_o^- = 0.5\,V \\ V_{in}^+ = V_{in}^- = 0\,V \end{cases}$$

a) $I_{ss} = I_{D1} + I_{D2} \Rightarrow \boxed{I_{ss} = 100\mu A}$

$$R_L = \frac{V_{DD} - V_{D1}}{I_{D1}} = \frac{2.5V - 0.5V}{50\mu A} \Rightarrow \boxed{R_L = 40k\Omega}$$

b) $$0.05\,mA = \left(\frac{1mA}{V^2}\right)\Big(\underbrace{V_{gs}}_{-V_s} - 0.3V\Big)^2\Big(1 + (0.1)^{\cdot 7}\underbrace{V_{Ds}}_{0.5 - V_s}\Big)$$

$\Rightarrow V_{gs} = V_g - V_s$, $V_g = 0V$ so $V_{gs} = -V_s$

$V_{Ds} = V_D - V_s$, $V_D = 0.5\,V \Rightarrow V_{Ds} = 0.5 - V_s$

$\Rightarrow V_s = -0.51\,V$ or $-\cancel{0.08}3\,V$, DC Voltages + currents marked in red, above

c) find g_m + g_{ds} $\qquad g_m = 2\left(\frac{\mu C_{ox} W_g}{2L_g}\right)(V_{gs} - V_{th})(1 + \lambda V_{Ds})$

$$g_m = 2\left(1\frac{mA}{V^2}\right)(0.51V - 0.3V)\left[1 + (0.1)(1.01V)\right], \quad g_m \equiv \frac{\partial I_D}{\partial V_{gs}}$$

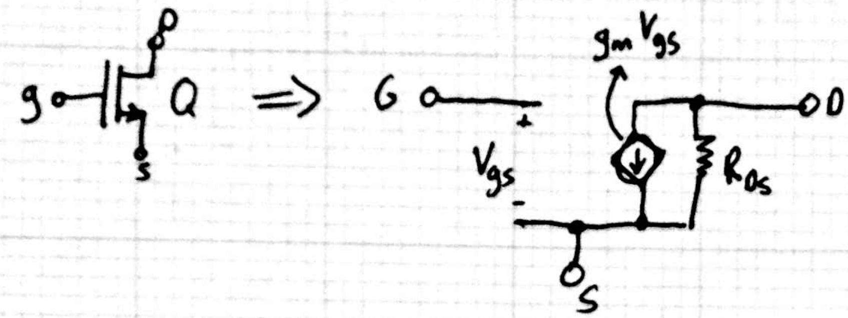
$$\Rightarrow \boxed{g_m = 0.462\ mS}$$

$$g_{ds} = \frac{1}{R_{ds}} \approx \lambda I_D = (0.1V^{-1})(50\mu A)$$

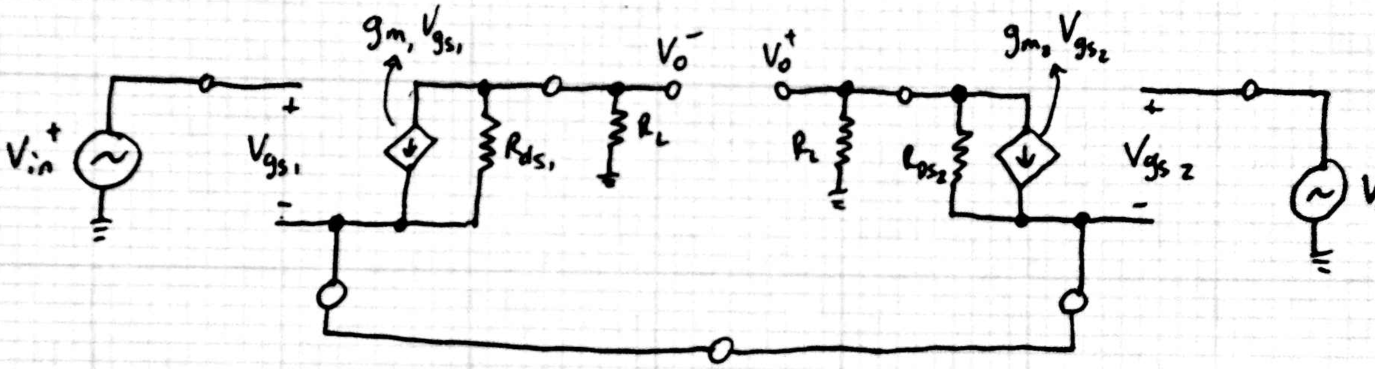
$$\Rightarrow \boxed{g_{ds} = 5\mu S} \Rightarrow R_{ds} = 200k\Omega$$

③ (con't)

d) small sig. equivs of $Q_1 + Q_2$ are identical due to symmetry, so:



e)



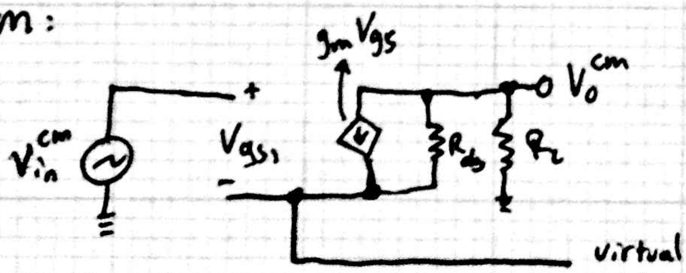
f) $V_{in}^+ = (1mV)\cos(2\pi \cdot 1kHz \cdot t)$, $V_{in}^- = (0.5mV)\cos(2\pi \cdot 1kHz \cdot t)$

NB: $V_{in}^+ + V_{in}^-$ are in phase,

$$V_{in}^d = V_{in}^+ - V_{in}^- = (0.5mV)\cos(2\pi \cdot 1kHz \cdot t)$$

$$V_{in}^{cm} = \frac{V_{in}^+ - V_{in}^-}{2} = (0.25mV)\cos(2\pi \cdot 1kHz \cdot t)$$

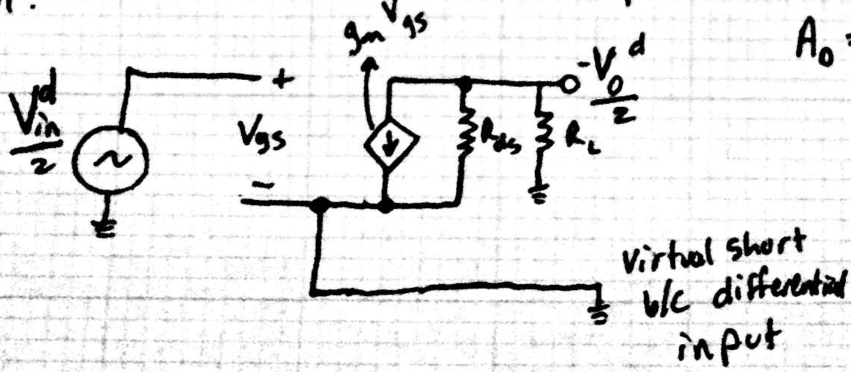
cm:



NB: $I_s = 0$ b/c of virtual open

$\therefore I_D = 0 \Rightarrow A_{cm} = \frac{V_o^{cm}}{V_{in}^{cm}} = 0$

Dm:



$$A_D = \frac{V_{out}^D}{V_{in}^d} = g_m (R_L \| R_{ds})$$

$$A_D = (0.462\,mS)\underbrace{(40k\Omega \| 200k\Omega)}_{33.3\,k\Omega}$$

$$A_D = 15.4$$

③ cont'd

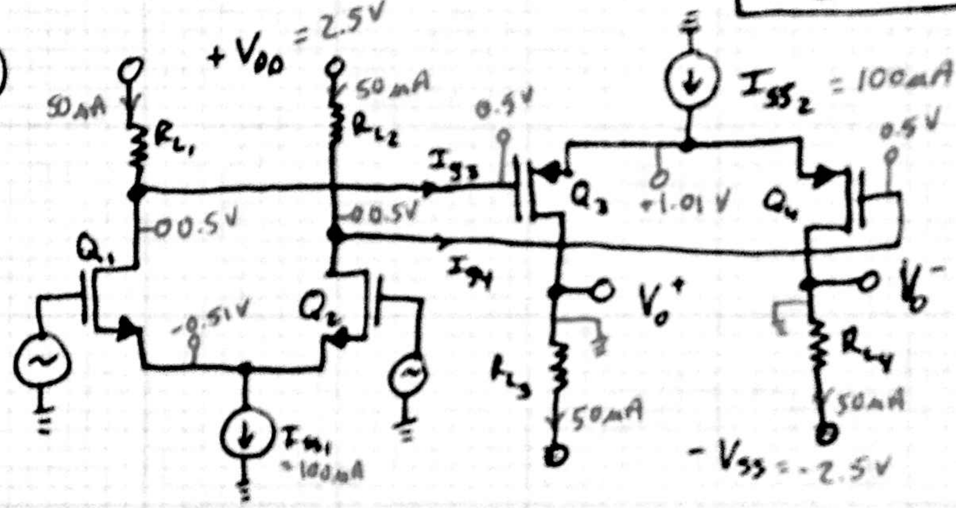
$$V_{out}^{cm} = A_{cm} V_{in}^{cm} = 0$$

$$V_{out}^{d} = A_d V_{in}^{d} = (15.4)(0.5\,mV)\cos(2\pi \cdot 1kHz \cdot t)$$

$$V_{out}^{d} = (7.7\,mV)\cos(2\pi \cdot 1kHz \cdot t)$$

$$V_o^+ = \frac{V_o^d}{2}, \quad V_o^- = -\frac{V_o^d}{2} \Rightarrow \boxed{\begin{aligned} V_o^+ &= (3.85\,mV)\cos(2\pi \cdot 1kHz \cdot t) \\ V_o^- &= -(3.85\,mV)\cos(2\pi \cdot 1kHz \cdot t) \end{aligned}}$$

④



$$\frac{\mu C_{ox} W_g}{2 L_g} = 1 \frac{mA}{V^2}$$

$\lambda = 0.1\,V^{-1}$

NFET: $V_{TN} = 0.3V$

PFET: $V_{TP} = -0.3V$

$$\left.\begin{aligned} V_{O_1} &= V_{O_2} = 0.5V \\ V_{O_3} &= V_{O_4} = 0.0V \\ I_{D_{1-4}} &= 50\mu A \end{aligned}\right\} DC$$

$-V_{SS} = -2.5V$

$V_{DD} = 2.5V$

a) we know $I_{g_3} = I_{g_4} = 0A$

so $I_{SS_1} = I_{D_1} + I_{D_2} \Rightarrow$ $I_{SS_1} =$ [redacted] 100 μA

$I_{SS_2} = I_{D_3} + I_{D_4} \Rightarrow$ $I_{SS_2} = 100\mu A$

$$R_{L_1} = R_{L_2} = \frac{V_{DD} - V_{O_{1,2}}}{I_{D_{1,2}}} = \frac{2.5V - 0.5V}{50\mu A} \Rightarrow \boxed{R_{L_1} = R_{L_2} = 40\,k\Omega}$$

$$R_{L_3} = R_{L_4} = \frac{V_{O_{3,4}} - V_{SS}}{I_{D_{3,4}}} = \frac{0V - 2.5V}{50\mu A} \Rightarrow \boxed{R_{L_3} = R_{L_4} = 50\,k\Omega}$$

b) 1st stage is identical to #3 DC bias condns

$$I_{D_{3,4}} = \left(\frac{1\,mA}{V^2}\right)\left(V_s - \overset{0.5V}{V_g} - 0.3V\right)^2\left[1 + (0.1V^{-1})(V_s - \overset{0}{V_o})\right] = 0.05\,mA$$

$\Rightarrow V_{S_3} = 1.013V$

④ cont

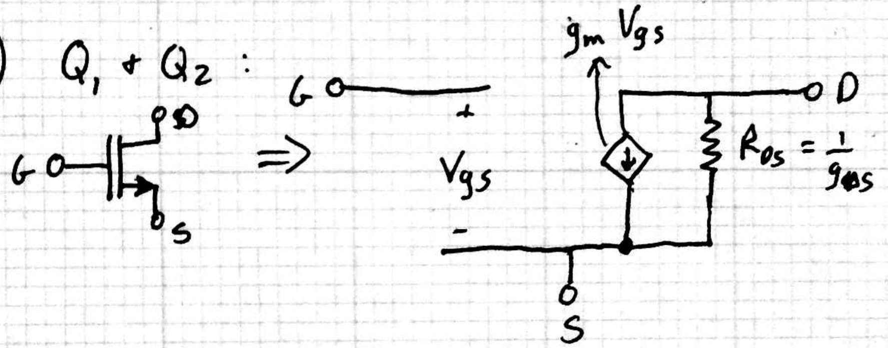
c) find g_m, g_{SO}, for $Q_1 + Q_2$: $\boxed{g_m = 0.462\,mS \quad g_{SO} = 5\,\mu S}$ from #3

$$g_m = 2\left(1\,\frac{mA}{V^2}\right)\left(\overset{1.013}{V_s} - \overset{0.5}{V_g} - 0.3\right)\left(1 + \lambda(1.01V)\right) = 0.462\,mS$$

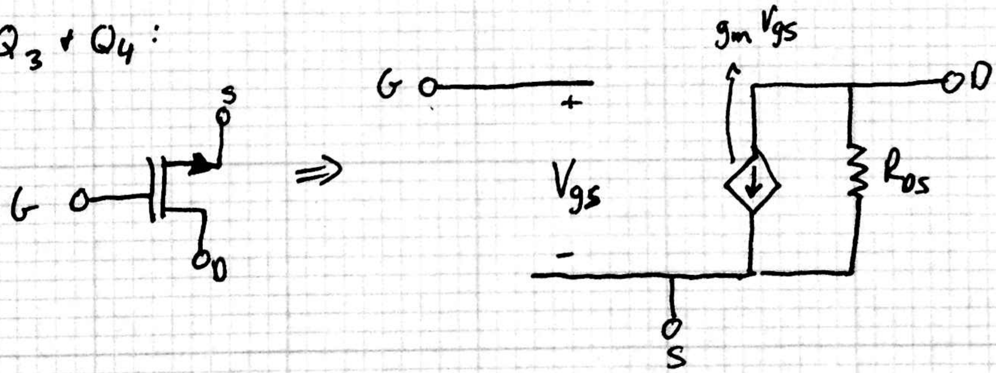
for $Q_3 + Q_4$: $\boxed{g_m = 0.462\,mS}$

$g_{SO} \approx \lambda I_0 = 5\,\mu S$ $\boxed{g_{OS} = 5\,\mu S}$

d) $Q_1 + Q_2$:



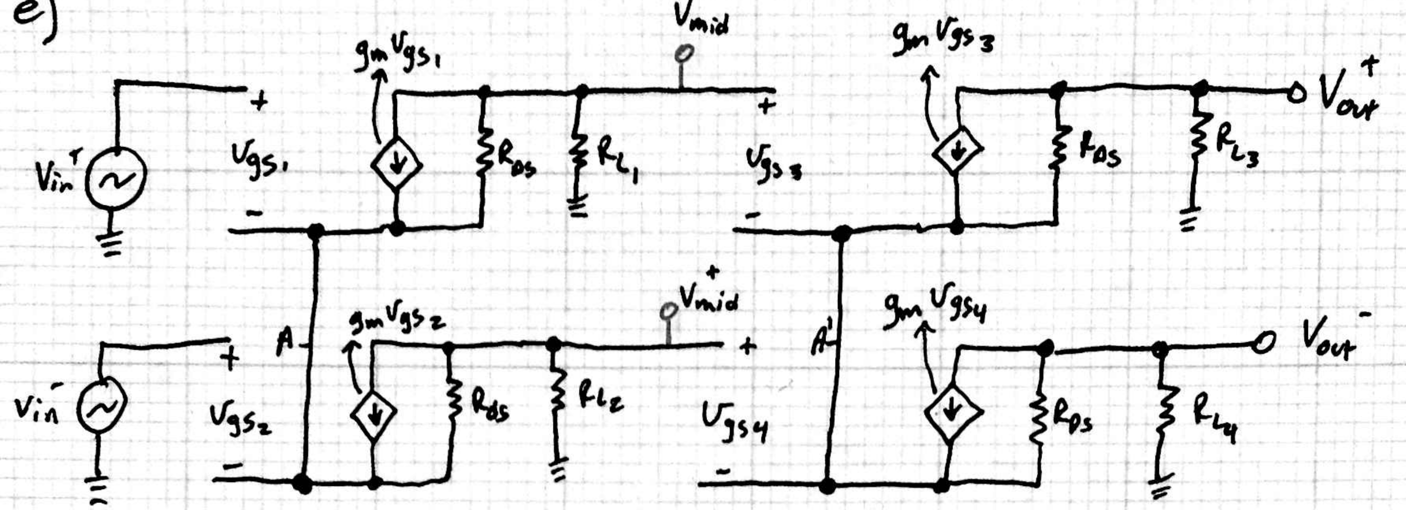
$Q_3 + Q_4$:



same as nmos b/c $V_{gs} < 0$ changes direction of controlled current source ST I_0 flows from Source → drain

④ con't

e)



F) $V_{in}^+ = (1\,mV)\cos(2\pi \cdot 1kHz \cdot t)$

$V_{in}^- = (0.5\,mV)\cos(2\pi \cdot 1kHz \cdot t)$

NB: $V_{mid}^{+,-}$ are equivalent to $V_o^{+,-}$ in problem #3

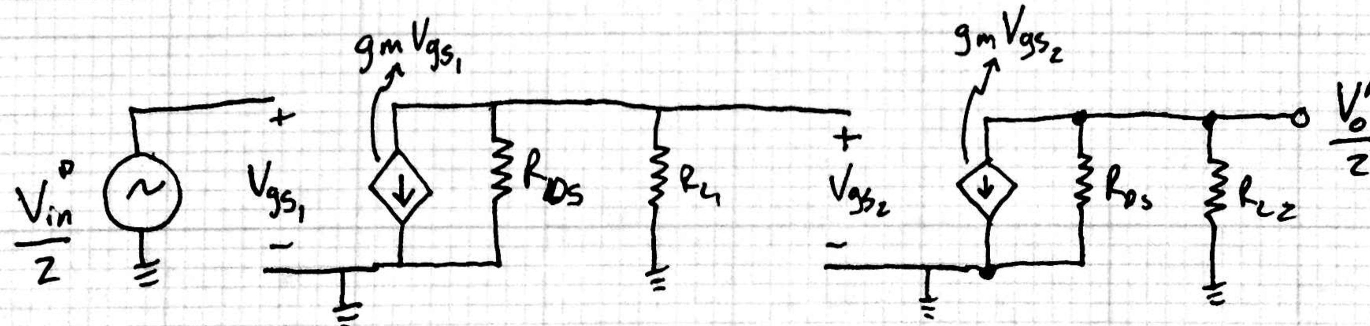
for Common mode input, A & A' are virtual open circuits

so clearly, as in #3, $\dfrac{V_o^{cm}}{V_{in}^{cm}} = 0$

for differential input:

b/c differential input is antisymmetric with the ckt symmetry, the voltages at A & A' must be zero, i.e. a virtual ground

so we consider only the left half ckt:



stage 1: $\dfrac{V_o^d}{V_{in}^d} = g_m(R_{DS} \| R_{L_1}) = A_{V_1} = 15.4$

$\Rightarrow A_{tot} = A_{V_1} A_{V_2}$

Stage 2: $\dfrac{V_o^d}{V_{in}^d} = g_m(R_{DS} \| R_{L_2}) = A_{V_2} = 18.48$

$\boxed{A_{tot} = 284.6}$

④ cont

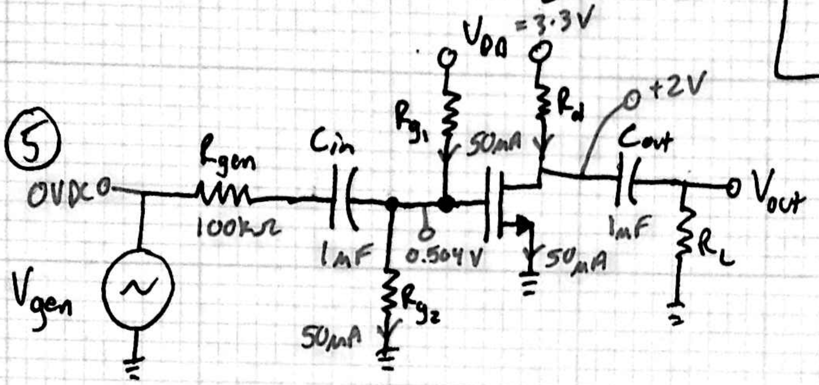
$$V_o^d = A_{tot} V_{in}^d = (284.6)(0.5\cos(2\pi\cdot 1kHz\cdot t))$$

$$V_o^d = (142\,mV)\cos(2\pi\cdot 1kHz\cdot t)$$

$$V_o^+ = \frac{V_o^d}{2}, \quad V_o^- = -\frac{V_o^d}{2} \Rightarrow$$

$$\boxed{\begin{aligned} V_o^+ &= (71\,mV)\cos(2\pi\cdot 1kHz\cdot t) \\ V_o^- &= -(71\,mV)\cos(2\pi\cdot 1kHz\cdot t)\end{aligned}}$$

⑤



$$\frac{\mu C_{ox} W_g}{2L_g} = \frac{1mA}{V^2}, \quad \lambda = 0.1\,V^{-1}, \quad V_{th} = 0.3V$$

$$I_d = 50\mu A, \quad I_{R_{g_1}} = 50\mu A$$

$$V_{DD} = 3.3V, \quad V_o = 2.0V$$

$$R_L = 4R_d, \quad R_{gen} = 100k\Omega$$

$$C_{in} = C_{out} = 1\mu F$$

a) $$I_D = \frac{\mu C_{ox} W_g}{2L_g}(V_{gs} - V_{th})^2(1 + \lambda V_{DS})$$

$$0.050\,mA = \left(\frac{1mA}{V^2}\right)(V_g - 0.3V)^2\left[1 + (0.1/V)(2.0V)\right]$$

$V_g = 0.504\,V$ so

$$V_g = V_{DD}\frac{R_{g2}}{R_{g_1} + R_{g2}} \quad \& \quad (R_{g_1} + R_{g2}) = \frac{V_{DD}}{I_{R_{g_1}}} = \frac{3.3V}{0.05mA} = 66k\Omega$$

$$0.504V = \frac{3.3V\ R_{g2}}{66k\Omega} \Rightarrow \boxed{R_{g2} = 10.08k\Omega \text{ so } R_{g_1} = 55.92k\Omega}$$

$$R_d = \frac{V_{DD} - V_o}{I_D} = \frac{3.3V - 2V}{0.05mA} \Rightarrow \boxed{R_d = 26k\Omega \text{ so } R_L = 104k\Omega}$$

b) see red labels, above

5 cont'd

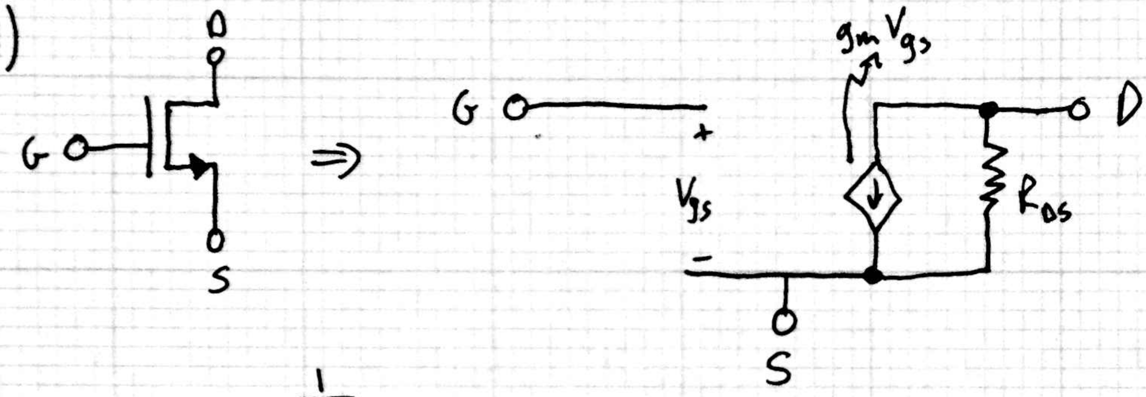
c) $$g_m = 2\left(\frac{\mu C_{ox} W_g}{2 L_g}\right)(V_{gs} - V_{th})(1 + \lambda V_{DS})$$

$$g_m = 2\left(1\,\frac{mA}{V^2}\right)(0.504\,V - 0.3\,V)\left[1 + (0.1\,V^{-1})(2.0\,V)\right]$$

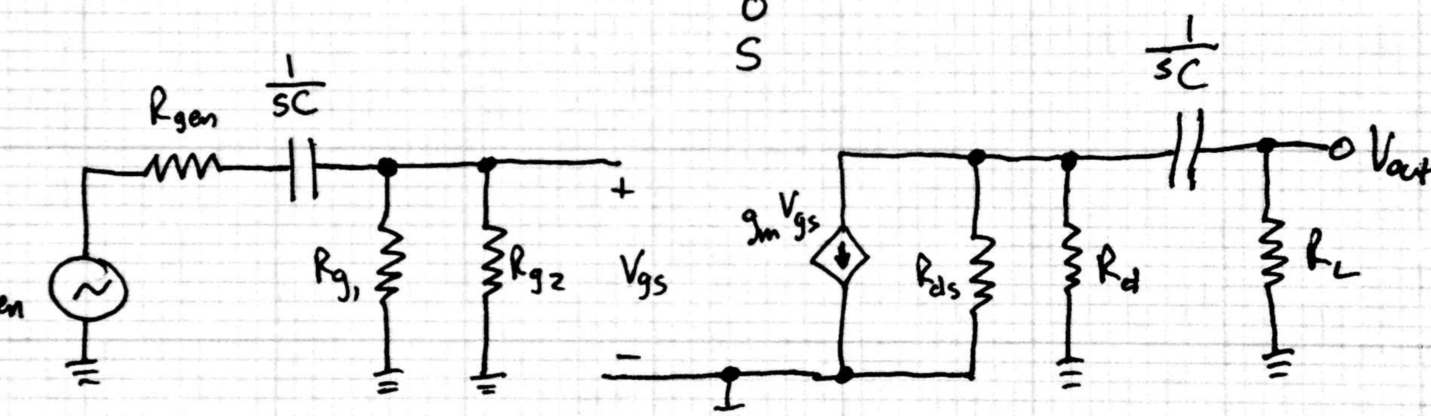
$$\Rightarrow \boxed{g_m = 0.4896\ mS}$$

$$g_{ds} = \frac{1}{R_{ds}} \cong \lambda I_D \quad \Rightarrow \quad \boxed{g_{ds} \approx 5\,\mu S}$$

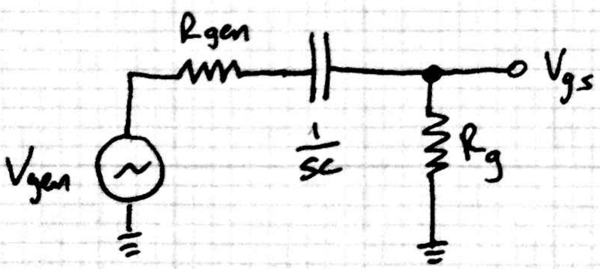
d)



e)



f) NB to find H(s) we can split the ckt in half:

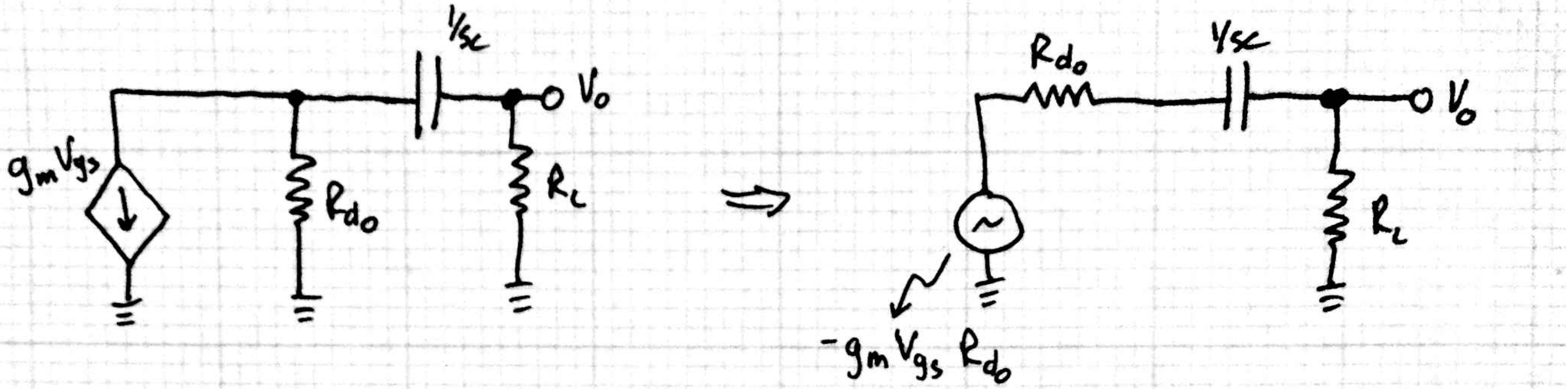


$$R_g = R_{g1} \parallel R_{g2}$$

$$\frac{V_{gs}}{V_{gen}} = \frac{R_g}{R_g + R_{gen}} \cdot \frac{sC(R_g + R_{gen})}{1 + sC(R_{gen} + R_g)}$$

5 cont

and for the right half, use Norton → Thevenin transform:



where $R_{do} = R_{ds} \| R_d$

so $$\frac{V_o}{V_{gs}} = -g_m R_{do} \left(\frac{R_L}{R_L + R_{do}}\right)\left(\frac{sC(R_L + R_{do})}{1 + sC(R_L + R_{do})}\right)$$

then $$H(s) = \frac{V_o}{V_{gen}} = \frac{V_o}{V_{gs}} \cdot \frac{V_{gs}}{V_{gen}}$$

$$\boxed{H(s) = -g_m R_{do}\left(\frac{R_g}{R_g + R_{gen}}\right)\left(\frac{R_L}{R_L + R_{do}}\right)\left(\frac{s\tau_1}{1 + s\tau_1}\right)\left(\frac{s\tau_2}{1 + s\tau_2}\right)}$$

where $\tau_1 \equiv C(R_g + R_{gen})$ + $\tau_2 \equiv C(R_g + R_{gen})$

$R_g \equiv R_{g_1} \| R_{g2}$, $R_{do} = R_{ds} \| R_d$

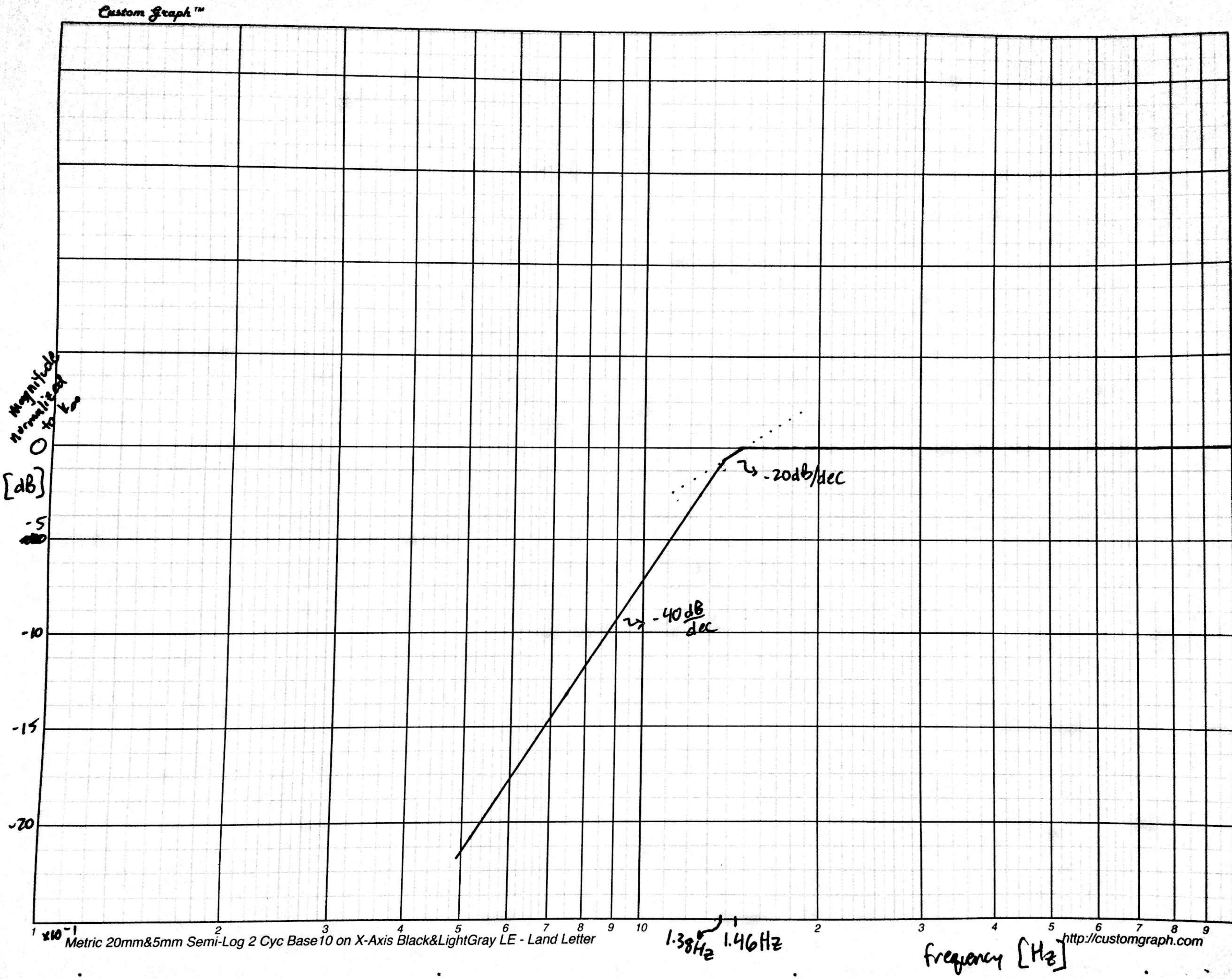
g) $\omega_1 \equiv \frac{1}{\tau_1}$ + $\omega_2 \equiv \frac{1}{\tau_2}$

so $$f_1 = \frac{1}{2\pi C(R_{g_1} \| R_{g2} + R_{gen})} = \frac{1}{2\pi(1\,\mu F)(10k\Omega \| 55.9k\Omega + 100k\Omega)}$$

$$\boxed{f_1 = 1.466\ Hz}$$

$$f_2 = \frac{1}{2\pi C(R_d \| R_{os} + R_L)} = \frac{1}{2\pi(1\,\mu F)(26k\Omega \| 20k\Omega + 104\,k\Omega)}$$

$$\boxed{f_2 = 1.380\ Hz}$$



Custom Graph™
Magnitude normalized to V_{∞}
0
[dB]
-5
-10
-15
-20
-20dB/dec
-40 dB/dec
1.38Hz
1.46Hz
1
×10⁻¹
2
3
4
5
6
7
8
9
10
Metric 20mm&5mm Semi-Log 2 Cyc Base10 on X-Axis Black&LightGray LE - Land Letter
http://customgraph.com
frequency [Hz]