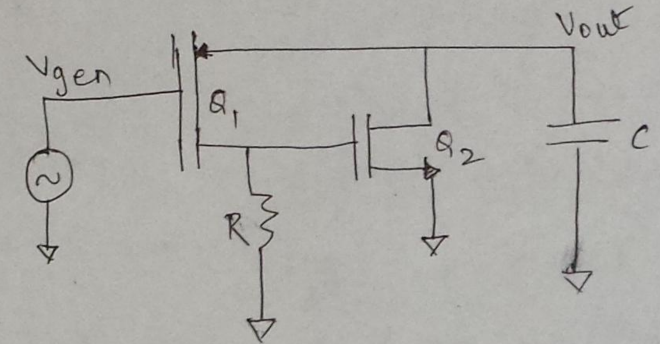
5/18/13

Homework - 6

1)

a)

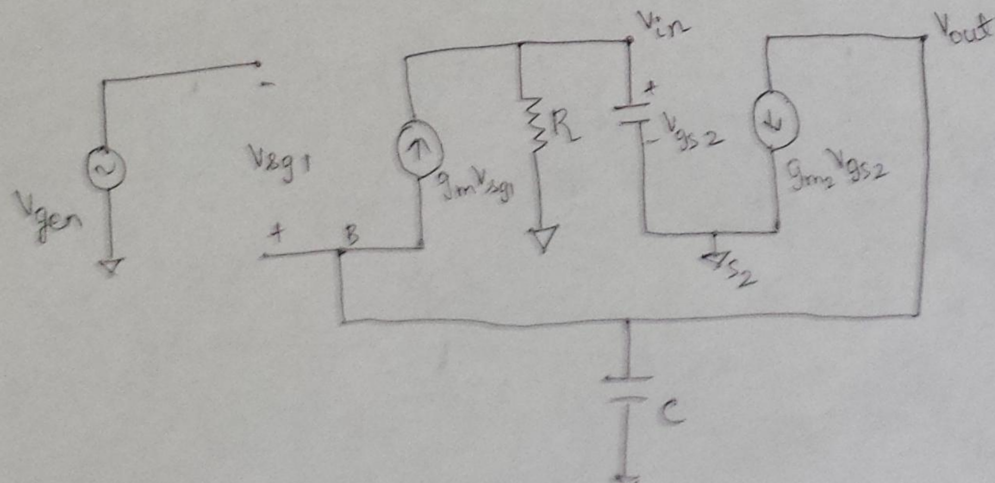


Q_1 : $C_{gs} = C_{gd} = 0$ R_{ds} infinity

Q_2 : $C_{gs} = 200fF$, $C_{gd} = 0$ R_{ds} = infinity

$g_{m1} = 100\,mS$, $g_{m2} = 200\,mS$, $R = 1000\,\Omega$ $C = 400\,fF$

Small Signal equivalent



b) $-V_{out} + V_{sg1} + V_{gen} = 0$

$$V_{sg1} = V_{out} - V_{gen} \quad \text{—(1)}$$

At B, write KCL

$$sCV_{out} + g_{m1} V_{sg1} + g_{m2} V_{gs2} = 0$$

$$g_{m1} [V_{out} - V_{gen}] + g_{m2} V_{gs2} + sC V_{out} = 0 \quad \text{—(2)}$$

At A write KCL

$$-g_{m1} V_{sg1} + \frac{V_{gs2}}{R} + sC_{gs2} V_{gs2} = 0$$

$$-g_{m1} [V_{out} - V_{gen}] + \frac{V_{gs2}}{R} + sC_{gs2} V_{gs2} = 0$$

$$V_{gs2} \left[\frac{1}{R} + sC_{gs2} \right] = g_{m1} [V_{out} - V_{gen}]$$

$$\boxed{V_{gs2} = \frac{g_{m1} R [V_{out} - V_{gen}]}{1 + sRC_{gs2}}} \rightarrow (3)$$

Substitute (3) in (2)

$$g_{m1} [V_{out} - V_{gen}] + \frac{g_{m2} [g_{m1} R (V_{out} - V_{gen})]}{1 + sRC_{gs2}} + sC V_{out} = 0$$

$\div V_{gen}$

$$\left(g_{m1} \frac{V_{out}}{V_{gen}} \right) - (g_{m1}) + \left(\frac{g_{m1} g_{m2} R}{1 + sRC_{gs2}} \right) \left[\frac{V_{out}}{V_{gen}} - 1 \right] + sC \frac{V_{out}}{V_{gen}} = 0$$

$$\frac{V_{out}}{V_{gen}}\left[g_{m1} + \frac{g_{m1}g_{m2}R}{1+sRC_{gs2}} + sC\right] = g_{m1} + \frac{g_{m1}g_{m2}R}{1+sRC_{gs2}}$$

$$\frac{V_{out}}{V_{gen}} = \frac{g_{m1} + \dfrac{g_{m1}g_{m2}R}{1+sRC_{gs2}}}{g_{m1} + \dfrac{g_{m1}g_{m2}R}{1+sRC_{gs2}} + sC}$$

$$= \frac{\dfrac{g_{m1}\left[1+sRC_{gs2}+g_{m2}R\right]}{1+sRC_{gs2}}}{g_{m1}\left[\dfrac{1+sRC_{gs2}+g_{m2}R}{1+sC_{gs2}R}\right] + sC}$$

$$= \frac{g_{m1}\left[1+sRC_{gs2}+g_{m2}R\right]}{g_{m1}\left[1+g_{m2}R\right]\left[1+\dfrac{s\left(\frac{C}{g_{m1}}+RC_{gs2}\right)+s^2RC}{1+g_{m2}R}\right]}$$

$$= \frac{\left(1+\dfrac{sRC_{gs2}}{1+g_{m2}R}\right)}{1+s\left(\dfrac{RC_{gs2}+\frac{C}{g_{m1}}}{1+g_{m2}R}\right)+s^2\left(\dfrac{C_{gs2}R\frac{C}{g_{m1}}}{1+g_{m2}R}\right)}$$

c) The denominator is of the form

$$1 + s\left(\frac{2\xi}{\omega_n}\right) + \frac{s^2}{\omega_n^2}$$

$$\omega_n^2 = \frac{1 + g_{m_2} R}{C_{gs2} R \left[\frac{C}{g_{m_1}}\right]}$$

$$\omega_n^2 = \frac{201}{8 \times 10^{-22}}$$

$$\omega_n = 5.012 \times 10^{11} \text{ rad/sec}$$

$$f_n = \frac{\omega_n}{2\pi} = 79.81 \text{ Ghz}$$

$$\xi = \frac{\omega_n}{2}\left[\frac{RC_{gs2} + \frac{C}{g_{m_1}}}{1 + g_{m_2} R}\right]$$

$$= \frac{5.012 \times 10^{11}}{2}\left[\frac{(1000 \times 200 \times 10^{-15}) + (4 \times 10^{-12})}{201}\right]$$

$$= 0.2543$$

d) Zeros

$$\omega_z = \frac{1 + g_{m_2} R}{R C_{gs2}} = 1.005 \times 10^{12}$$

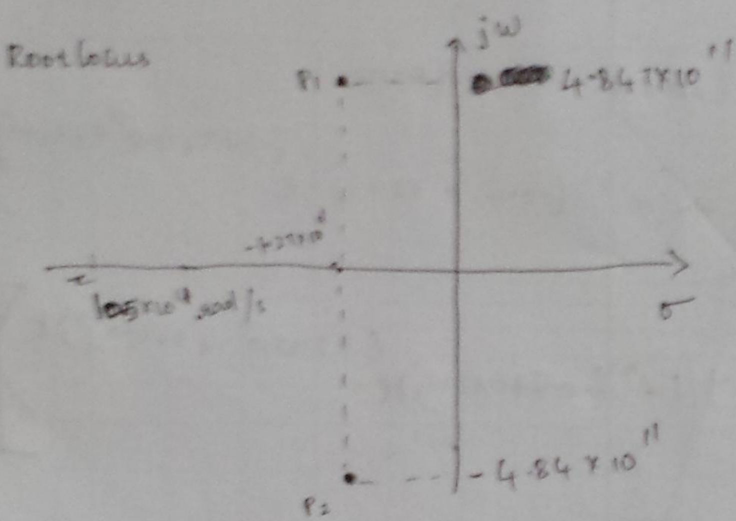
$$f_z = 1.6 \times 10^{11} \text{ Hz}$$

It has two poles at

$$\omega_{p1} = -1.27\times10^{11} + j\,4.847\times10^{11}$$

$$\omega_{p2} = -1.27\times10^{11} - j\,4.8477\times10^{11}$$

Root locus



e) Plot attached.

f) $V_{gen}(s) = \frac{10^{-3}}{s}$

$$V_{out}(s) = \frac{\left(\frac{10^{-3}}{s}\right)\left[1 + s\times10^{-12}\right]}{1 + 1.0147\times10^{-12}s + s^2\left(3.98\times10^{-24}\right)}$$

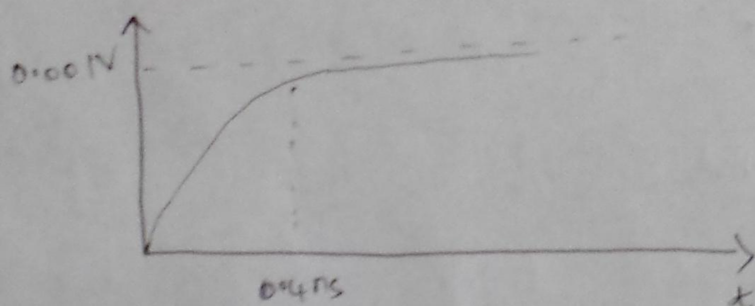
$$V_{out}(s) = \frac{10^{-3} + 10^{-15}\times s}{s + 1.0147\times10^{-12}s^2 + s^3\left(3.98\times10^{-24}\right)}$$

Separate using partial fractions technique

$$V_{out}(s) = \frac{10^{-3}}{s} + \frac{10^{-3}\left[-0.5-0.1227j\right]}{s-\left(-1.2747\times10^{11}+4.8477\times10^{11}j\right)} + \frac{10^{-3}\left[-0.5+0.1227j\right]}{s-\left(-1.2747\times10^{11}-4.8477j\times10^{11}\right)}$$

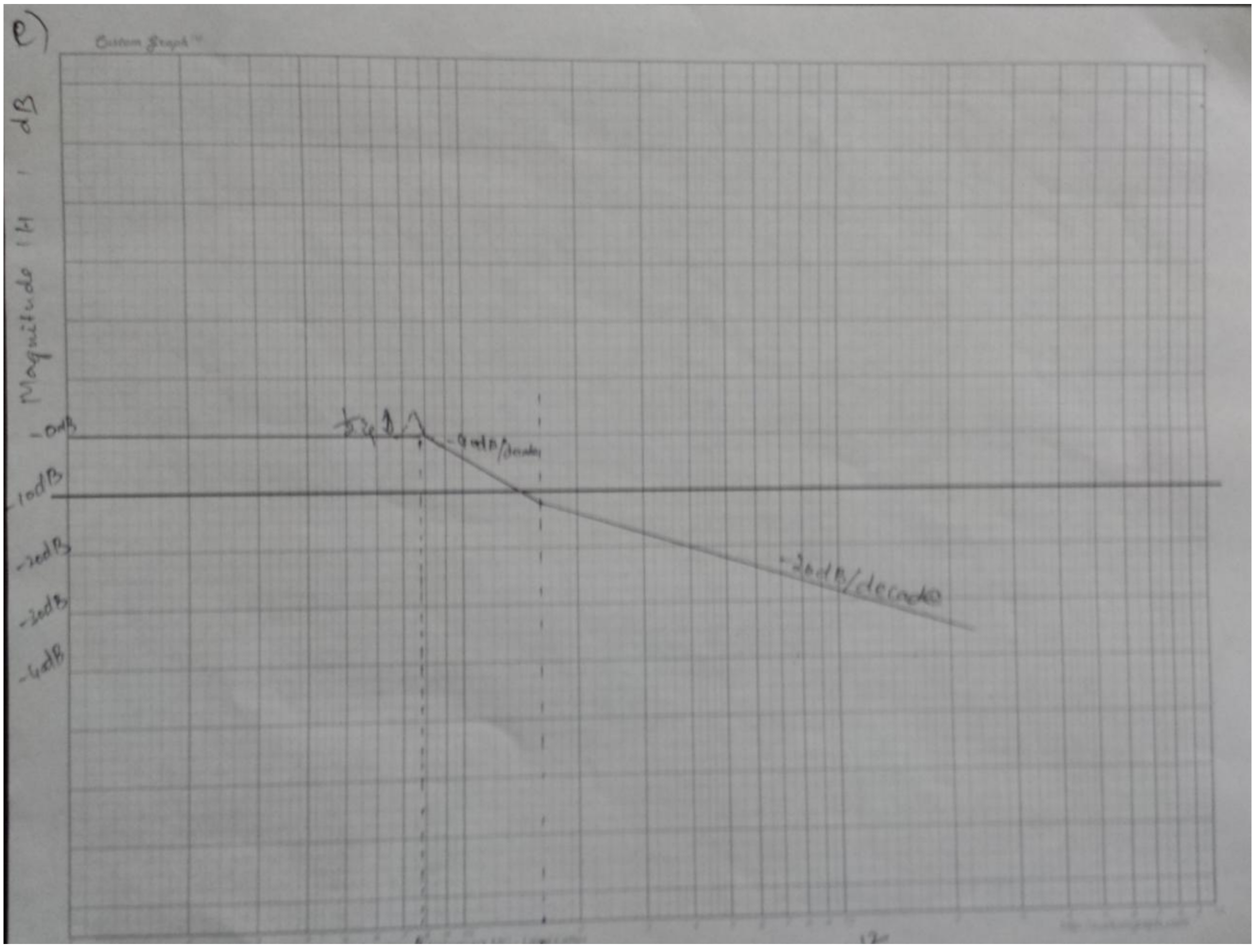
Take Inverse Laplace Transform

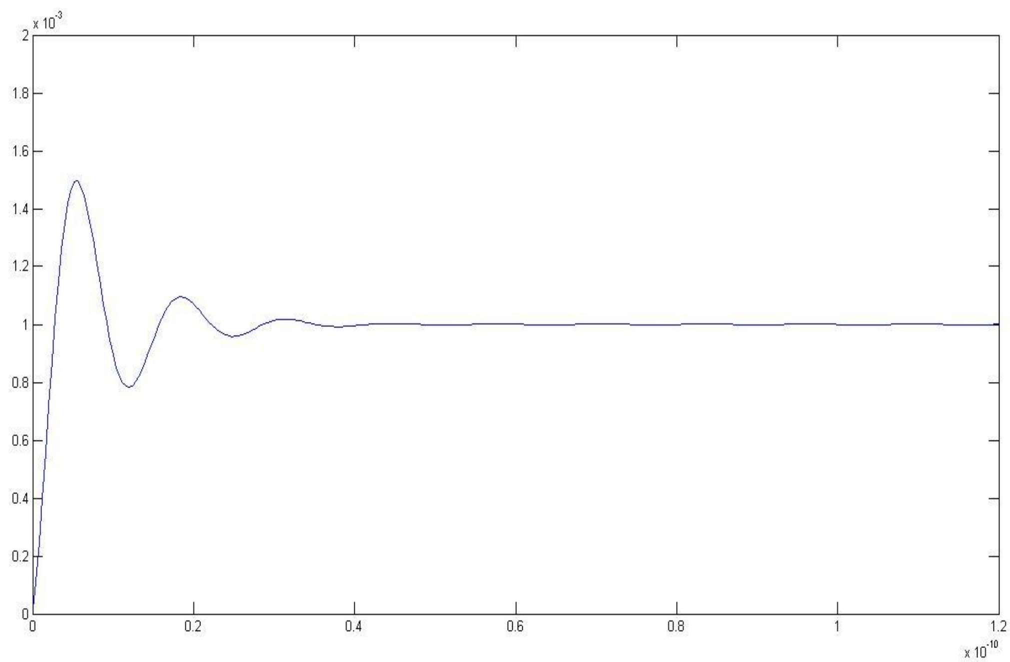
$$V_{out}(t) = u(t)\Big[0.001 + \left((10^{-3})(-0.5-0.1227j)\,e^{(-1.2747\times10^{11}+4.84\times10^{11}j)t}\right)$$
$$+\left((10^{-3})(-0.5+0.1227j)\,e^{(-1.274\times10^{11}-4.84\times10^{11}j)t}\right)\Big]$$



⇓

See the plot attached.





Output waveform

④

Problem 2

From the graph, System has three zeros at origin and 6 poles.

Poles are located

$\omega_{p1} = -0.3\times10^9 + j(2.6\times10^9)$

$\omega_{p2} = -0.3\times10^9 - j(2.6\times10^9)$

$\omega_{p3} = -0.3\times10^9 + j(3.4\times10^9)$

$\omega_{p4} = -0.3\times10^9 - j(3.4\times10^9)$

$\omega_{p5} = -0.5\times10^9 + j(3\times10^9)$

$\omega_{p6} = -0.5\times10^9 + j(3\times10^9)$

Write equation in terms of $H(s) = \dfrac{ks^3}{(s-p_1)(s-p_2)\ldots}$ to calculate k

$$H(s) = \frac{k\, s^3}{(s^2 + 6\times10^8 s + 6.85\times10^{18})(s^2 + 6\times10^8 s + 1.165\times10^{19})(s^2 + 10^9 s + 9.25\times10^{18})}$$

When $s = j(3\times10^9)$, $H(s) = 1$

$$H(s) = \frac{k\,(j\omega)^3}{[6.85\times10^{18} - \omega^2 + j\,6\times10^8\omega][1.165\times10^{19} - \omega^2 + j\,6\times10^8\omega][9.25\times10^{18} - \omega^2 + j\,10^9\omega]}$$

$$(-j\omega)^3 K = \left[-2.15\times10^{18} + j\,1.8\times10^{18}\right]\left[2.65\times10^{18} + j\,1.8\times10^{18}\right]\left[2.5\times10^{17} + 3\times10^{18} j\right]$$

Take Modulus on both sides

$$\omega^3 K = \left(2.80\times10^{18}\right)\left(3.203\times10^{18}\right)\left(3.01\times10^{18}\right)$$

$$K = 1.000\times10^{+27}$$

$$H(s) = \frac{K s^3}{A\left(1-\frac{s}{\omega_{p1}}\right)\left(1-\frac{s}{\omega_{p2}}\right)\left(1-\frac{s}{\omega_{p3}}\right)\left(1-\frac{s}{\omega_{p4}}\right)\left(1-\frac{s}{\omega_{p5}}\right)\left(1-\frac{s}{\omega_{p6}}\right)}$$

where $A = \omega_{p1}\,\omega_{p2}\,\omega_{p3}\,\omega_{p4}\,\omega_{p5}\,\omega_{p6}$

b) To plot the Bode plot

Calculate the three quadratic pole frequencies.

$$s^2 + 6\times10^{8} s + 6.85\times10^{18} = 6.85\times10^{18}\left[\frac{s^2}{6.85\times10^{18}} + \frac{6\times10^{8}}{6.85\times10^{18}} s + 1\right]$$

$$= 6.85\times10^{18}\left[1 + \frac{2\xi_1}{\omega_n} s + \frac{s^2}{\omega_n^2}\right]$$

where $\omega_{n_1}^2 = 6.85\times10^{18}$, $\omega_{n_1} = 2.61\times10^{9}$ rad/sec $f_{n_1} = 416.15$ Mhz

(2)

Similarly calculate the others two pole frequencies

$$s^2 + 6\times10^8 s + (1.165\times10^{19}) = 1.165\times10^{19}\left(1 + 5.1502\times10^{-11} s + \frac{s^2}{1.165\times10^{19}}\right)$$

$$\therefore \omega_{n_2}^2 = 1.165\times10^{19}$$

$$\omega_{n_2} = 3.413\times10^9 \text{ rad/sec}$$

$$f_{n_2} = 543.47\text{ MHz}$$

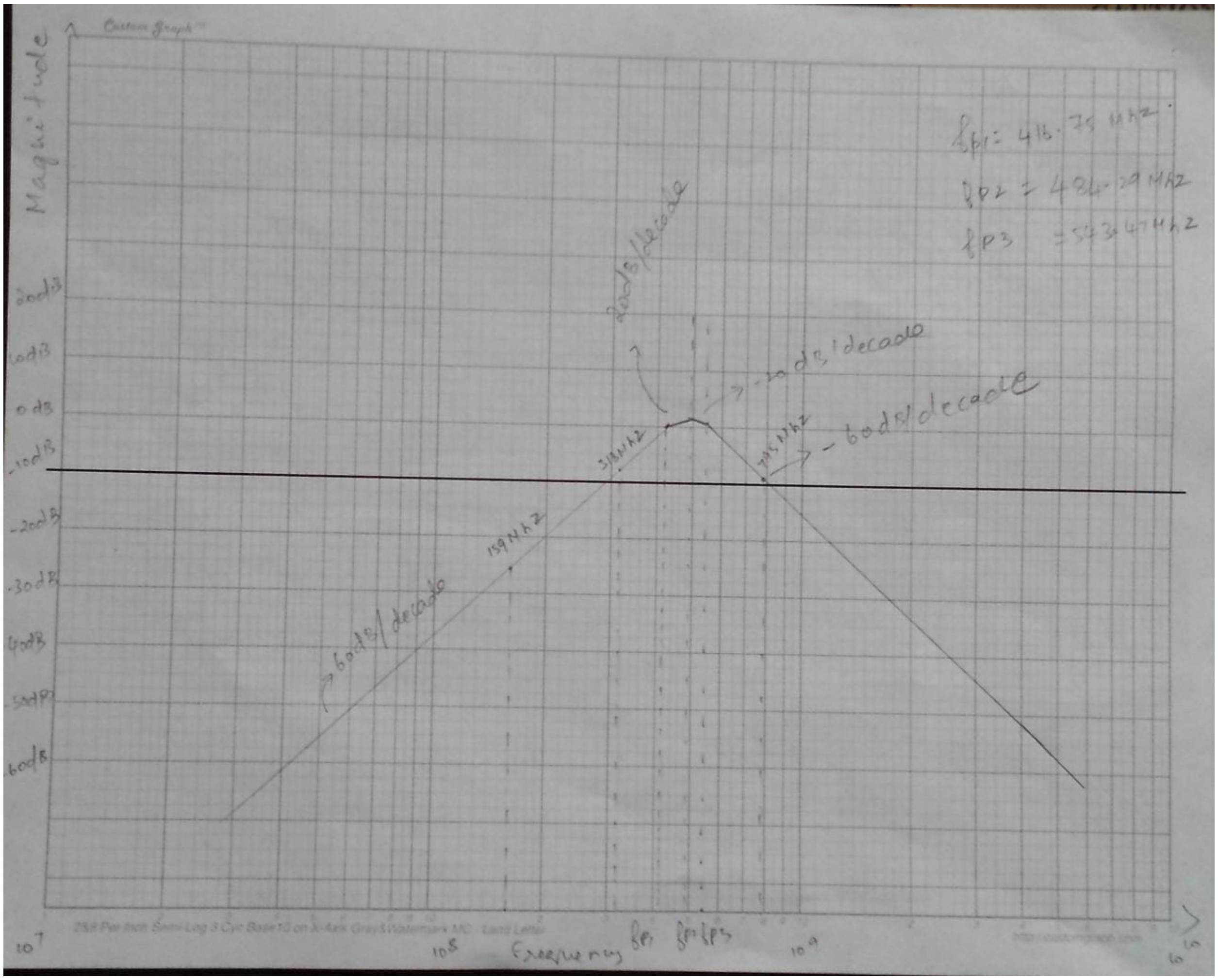
$$s^2 + 10^9 s + 9.25\times10^{18} = 9.25\times10^{18}\left[1 + 1.08\times10^{-10} s + \frac{s^2}{9.25\times10^{18}}\right]$$

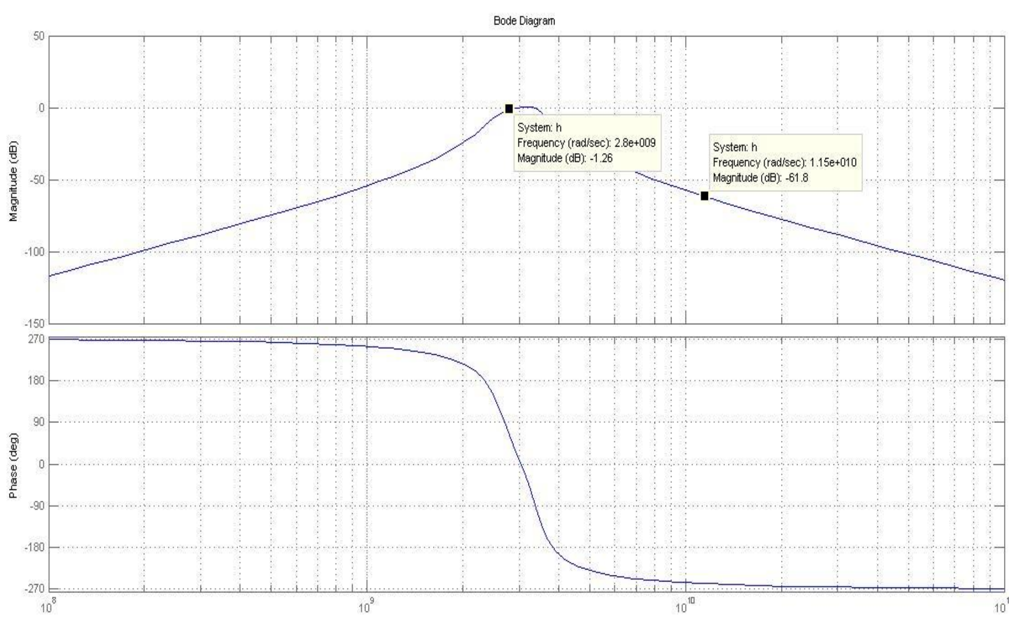
$$\omega_{n_3}^3 = 9.25\times10^{18}$$

$$\omega_{n_3} = 3.041\times10^9$$

$$f_{n_3} = 484.29\text{ MHz}$$

$$\therefore H(s) = \frac{1.3546\times10^{-30}\, s^3}{\left(1 + 8.759\times10^{-11} s + 1.459\times10^{-19} s^2\right)\left(1 + 5.15\times10^{-11} s + 8.583\times10^{-20} s^2\right) + \left[1 + 1.08\times10^{-10} s + 1.081\times10^{-19} s^2\right]}$$


Magnitude
$f_{P1} = 416.75$ MHz
$f_{P2} = 484.29$ MHz
$f_{P3} = 543.47$ MHz
-20dB/decade
-60dB/decade
60dB/decade
60dB/decade
159MHz
318MHz
795MHz
Frequency


Bode Diagram
System: h
Frequency (rad/sec): 2.8e+009
Magnitude (dB): -1.26
System: h
Frequency (rad/sec): 1.15e+010
Magnitude (dB): -61.8
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)

Problem 3 ①

$R_{rad} = 50\,\Omega$

$$\left(\frac{\mu C_{ox} W_g}{2L_g}\right) = 1\ \frac{mA}{V^2}$$

$\lambda = 0.05\,V^{-1}$

$V_{th} = 0.3\,V$

a) $$I_d = \left(\frac{\mu C_{ox} W_g}{2L_g}\right)\left(V_{gs} - V_{th}\right)^2 (1 + \lambda V_{DS})$$

Neglect λV_{ds}

$$0.1\,mA = 1\,mA/V^2 \,[V_{in} - 0.3]^2$$

$$0.1 = [V_{in} - 0.3]^2$$

$$V_{in}^2 + 0.09 - 0.6 V_{in} = 0.1$$

$$V_{in}^2 - 0.6 V_{in} - 0.01 = 0$$

$$V_{in} = 0.616\,V$$

b) $V_{DD} = 3.3\,V$

$V_D = 1.5\,V$

$$R_L = \frac{V_{DD} - V_D}{0.1\,mA}$$

$$= \frac{1.8}{0.1\,mA} = 18\,k\Omega$$

c) $g_m = \frac{\partial I_d}{\partial V_{gs}}$

$= \left(\frac{\mu C_{ox} W_g}{2L_g}\right)\left[1+\lambda V_{ds}\right]\left[2(V_{gs}-V_{th})\right]$

$= \frac{1mA}{V^2}\left[1+(0.05\times1.5)\right]\left[2(0.616-0.3)\right]$

$= \frac{1mA}{V^2} \times 1.075 \times 0.632$

$= 0.679\,mS$

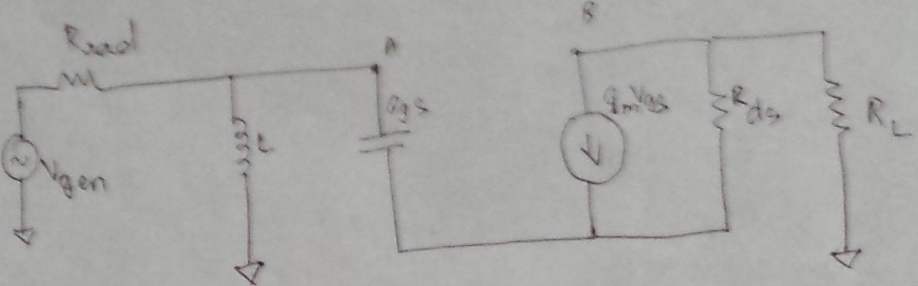
$g_{ds} = \frac{\partial I_D}{\partial V_{DS}} = \left(\frac{\mu C_{ox} W_g}{2L_g}\right)\left(V_{gs}-V_{th}\right)^2 [\lambda]$

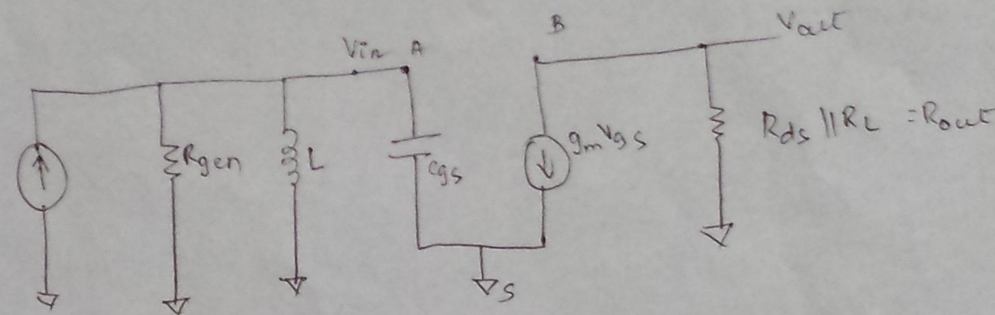
$= \frac{1mA}{V^2} \times 0.0998 \times 0.05$

$= 4.992 \times 10^{-6}$

$R_{ds} = 2.002 \times 10^{5}\,\Omega$

d)





Write KCL at node A

$$\frac{V_{in}}{R_{gen}} - \frac{V_{gen}}{R_{gen}} + V_{in}\, sC_{gs} + \frac{V_{in}}{sL} = 0$$

$$V_{in}\left[G_{gen} + \frac{1}{sL} + sC_{gs}\right] = \frac{V_{gen}}{R_{gen}}$$

$$V_{in} = \frac{V_{gen}}{1 + \frac{R_{gen}}{sL} + sR_{gen}C_{gs}} \quad - (1)$$

Write KCL at B

$$g_m V_{gs} + \frac{V_{out}}{R_{out}} = 0$$

$$g_m V_{in} + \frac{V_{out}}{R_{out}} = 0 \quad (2)$$

Substitute (1) in (2)

$$\frac{g_m V_{gen}}{1 + \frac{R_{gen}}{sL} + sR_{gen}C_{gs}} + \frac{V_{out}}{R_{out}} = 0$$

$$\left(\frac{V_{out}}{V_{gen}}\right)\left(\frac{1}{R_{out}}\right) = \frac{-g_m}{1 + \frac{R_{gen}}{sL} + sR_{gen}C_{gs}}$$

$$\frac{V_{out}}{V_{gen}} = \frac{-g_m R_{out}}{1 + \frac{R_{gen}}{sL} + sR_{gen}C_{gs}}$$

$R_{out} = 18k\Omega \parallel (2\times10^5)$

$= 16.513\,k\Omega$

$R_{gen} = 50\Omega$

$L = 0.797\ nH$

$C_{gs} = 31.8\ pF$

(3)

$$\frac{V_{out}}{V_{gen}} = \frac{(-g_m R_{out})\, sL}{sL + R_{gen} + s^2 L R_{gen} C_{gs}}$$

$$= \frac{(-g_m R_{out} L)\, s}{R_{gen}\left[1 + s\left(\frac{L}{R_{gen}}\right) + s^2 (L C_{gs})\right]}$$

$$= \frac{k \quad s}{1 + sa + s^2 b}$$

$$k = \frac{-g_m R_{out} L}{R_{gen}} \qquad \left(\frac{A}{V} \times \Omega \cdot s\right)$$

$$(s)$$

$$= -1.78 \times 10^{-10}$$

$$a = \frac{L}{R_{gen}} = 1.594 \times 10^{-11}$$

$$b = L C_{gs} = 2.5344 \times 10^{-20}$$

The two poles occur at

$$s_{p1} = -0.313\times10^{9} + 6.273\times10^{9} j$$

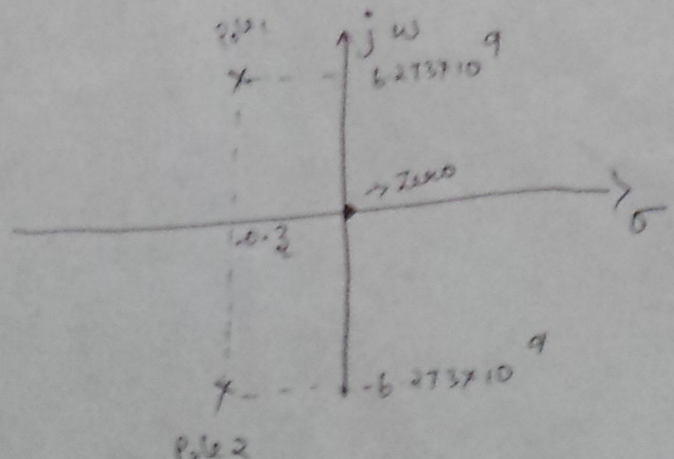
$$s_{p2} = -0.313\times10^{9} - 6.273\times10^{9} j$$

$$\therefore H(s) = \frac{k\, s}{-s_{p1}\left(1 - \frac{s}{s_{p1}}\right)(-s_{p2})\left(1 - \frac{s}{s_{p2}}\right)}$$

$$= \frac{k\, s}{(s_{p1} s_{p2})\left(1 - \frac{s}{s_{p1}}\right)\left(1 - \frac{s}{s_{p2}}\right)}$$

$$= \frac{k}{(3.944\times10^{19})} \frac{s}{\left(1 - \frac{s}{s_{p1}}\right)\left(1 - \frac{s}{s_{p2}}\right)}$$

(f)



g) Plot Attached (4)

h) $$\frac{V_{out}}{V_{gen}} = \frac{-(-1.78\times10^{-10})\,s}{1+s(1.594\times10^{-11})+s^2(2.544\times10^{-20})}$$

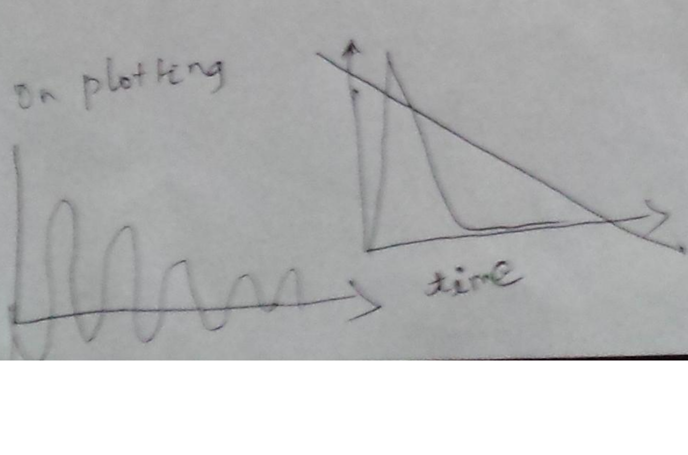
$$V_{gen}(s) = \frac{10^{-3}}{s}$$

$$V_{out}(s) = \frac{\left(\frac{-10^{-3}}{s}\right)(1.78\times10^{-10})\,s}{1+s(1.594\times10^{-11})+s^2(2.544\times10^{-20})}$$

$$= \frac{0.5599\times10^{-3}j}{s-(-0.3138\times10^{9}+6.2667\times10^{9}j)} - \frac{0.5599\times10^{-3}j}{s-(-0.3138\times10^{9}-6.2667\times10^{9}j)}$$

$$V_{out}(t) = (0.5599\times10^{-3}j)\left[e^{(-0.3138\times10^{9}+6.26\times10^{9}j)t} - e^{(-0.313\times10^{9}-6.266\times10^{9}j)t}\right]$$

On plotting



=> Plot attached

5)

For the Bode plot $H(s)$ is of the form

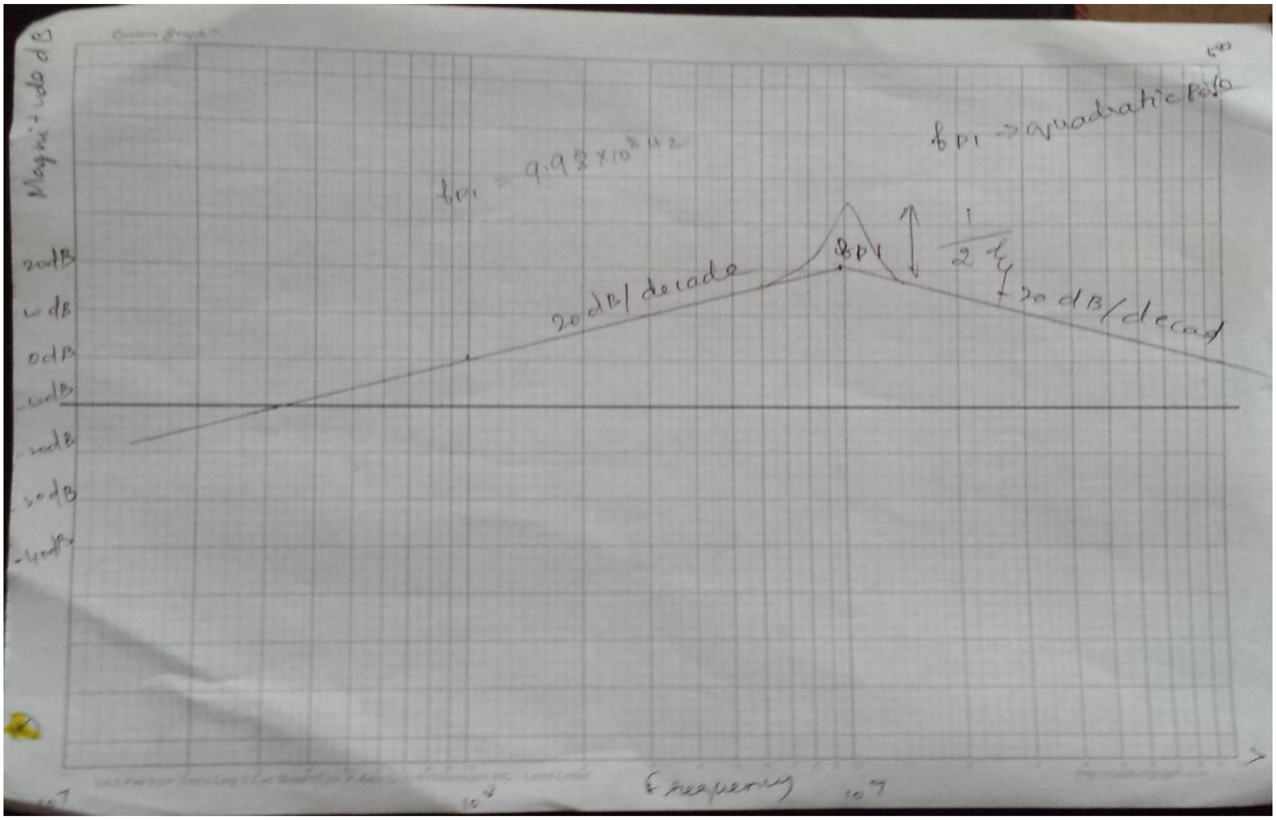
$$H(s) = K \frac{s \frac{2\xi}{\omega_n}}{\frac{s^2}{\omega_n^2} + \left(\frac{2\xi s}{\omega_n}\right) + 1}$$

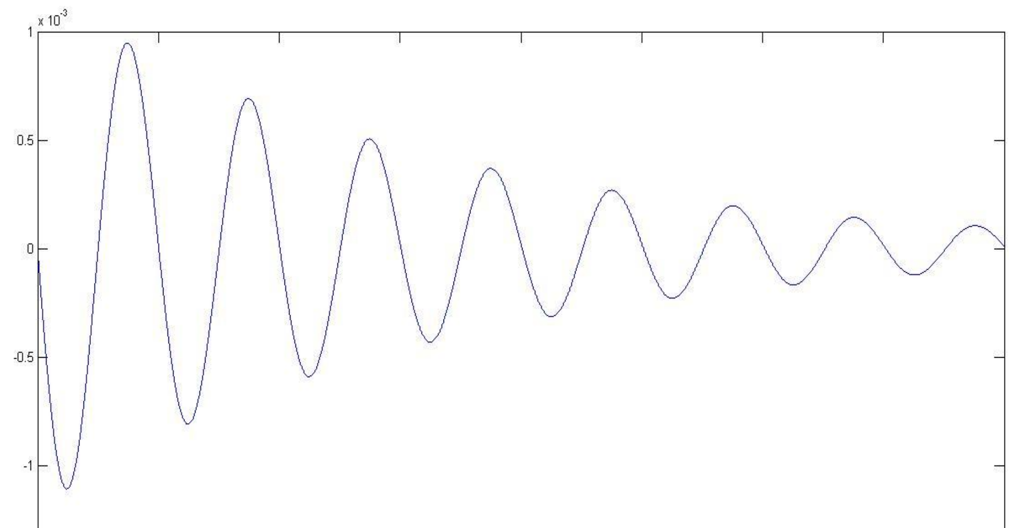
In this case $K = 11.166$

Quadratic pole at $\omega_n = 6.269 \times 10^9$ rad/sec

Zero at the origin.

$f_n = 9.98 \times 10^8$ Hz





Output response