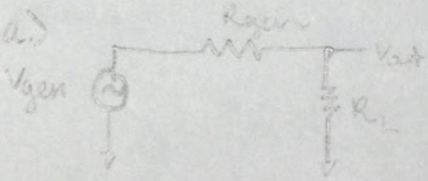
ECE 2C Homework #7

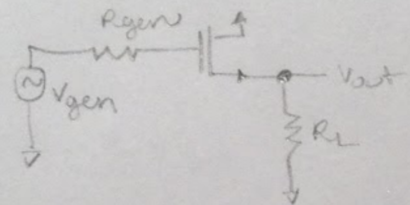
1.) a.)



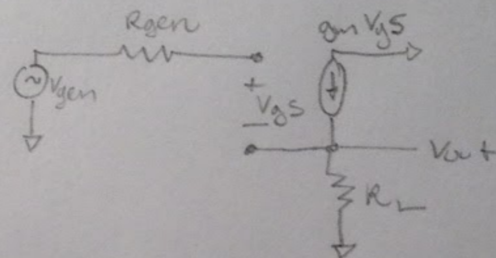
$$V_{out} = \frac{R_L V_{gen}}{R_L + R_{gen}}$$

$$\frac{V_{out}(s)}{V_{gen}(s)} = \frac{R_L}{R_L + R_{gen}}$$

b.)



$\longrightarrow$



$$V_{gs} = V_{gen} - V_{out}$$

$$V_{out} = g_m(V_{gen} - V_{out}) R_L \qquad \frac{V_{out}}{V_{gen}} = \frac{g_m R_L}{(1 + g_m R_L)}$$

$$V_{out}(1 + g_m R_L) = g_m V_{gen} R_L$$

c.) from a)

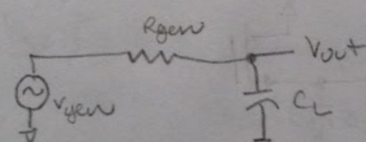
$$\frac{V_{out}}{V_{gen}} = \frac{1K\Omega}{1K\Omega + 1M\Omega} \sim 10^{-3}$$

from b)

$$\frac{V_{out}}{V_{gen}} = \frac{1mS \cdot 1K\Omega}{(1 + 1mS \cdot 1k\Omega)} = \frac{1}{2}$$

so circuit ii gives a larger output signal

d.)

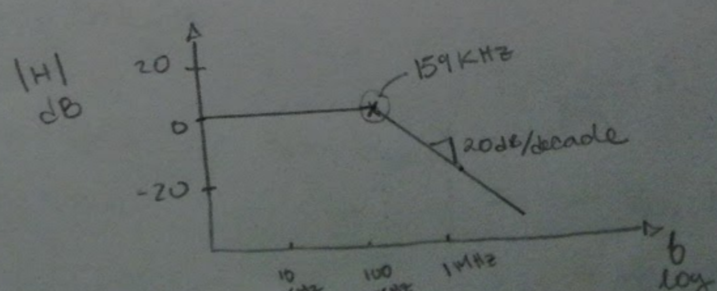


$$V_{out}(s) = \frac{\frac{1}{C_L s} V_{gen}}{R_{gen} + \frac{1}{C_L s}} \Rightarrow \frac{V_{out}}{V_{gen}} = \frac{1}{R_{gen} C_L s + 1}$$

e.) $\tau = R_{gen} C_L = 1M\Omega \cdot 1pF = 1\mu s$

$$\frac{V_{out}}{V_{gen}} = \frac{1}{1 + s\tau}$$

$$f_P = -\frac{1}{2\pi\tau} = 159\,KHz$$



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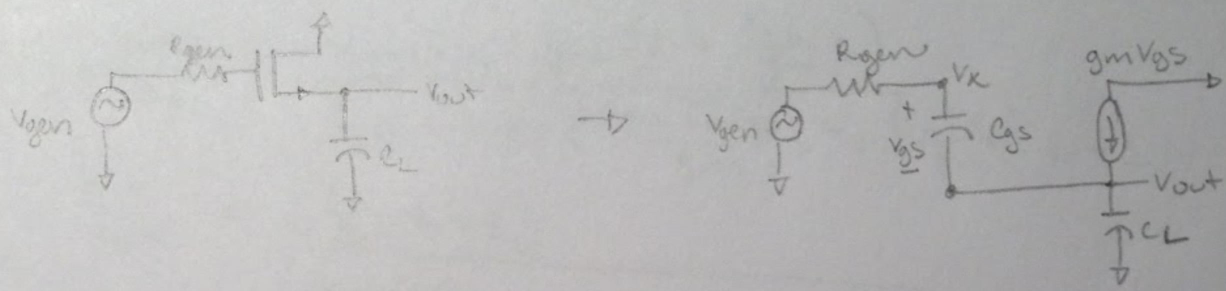
1d (2nd)



$$V_x = V_{gs} + V_{out}$$

@ node V_x $\Sigma i = 0$

$$(V_{gen} - V_x)\,G_{gen} = (V_x - V_{out})\,C_{gs}S$$

$$V_{gen}G_{gen} = V_x G_{gen} + (V_x - V_{out})\,C_{gs}S$$

$$V_{gen}G_{gen} = (V_{gs} + V_{out})\,G_{gen} + (V_{gs} + \cancel{V_{out}} - \cancel{V_{out}})\,C_{gs}S$$

$$V_{gen}G_{gen} = V_{gs}\,(G_{gen} + C_{gs}S) + V_{out}(G_{gen})$$

@ node V_{out} $\Sigma i = 0$

$$(V_x - V_{out})\,C_{gs}S + g_m V_{gs} = V_{out}C_L S$$

$$V_{gs}C_{gs}S + g_m V_{gs} = V_{out}C_L S$$

$$0 = V_{gs}(C_{gs}S + g_m) + V_{out}(-C_L S)$$

$$\begin{vmatrix} G_{gen} + C_{gs}S & G_{gen} \\ C_{gs}S + g_m & -C_L S \end{vmatrix} \begin{bmatrix} V_{gs} \\ V_{out} \end{bmatrix} = \begin{bmatrix} V_{gen}G_{gen} \\ 0 \end{bmatrix}$$

$$D(s) = -C_L C_{gs}S^2 - C_L G_{gen}S - C_{gs}G_{gen}S - G_{gen}g_m$$

$$= -g_m G_{gen}\left(\frac{R_{gen}C_L C_{gs}}{g_m}S^2 + \frac{(C_L + C_{gs})}{g_m}S + 1\right)$$

$$N(s) = \begin{vmatrix} G_{gen} + C_{gs}S & V_{gen}G_{gen} \\ C_{gs}S + g_m & 0 \end{vmatrix} = -V_{gen}G_{gen}(C_{gs}S + g_m)$$

$$= -V_{gen}G_{gen}\,g_m\left(\frac{C_{gs}}{g_m}S + 1\right)$$

d (2nd d) continued

$$V_{out}(s) = \frac{N(s)}{D(s)} = \frac{-V_{gen} G_{gen} g_m \left(1 + \frac{C_{gs}}{g_m} s\right)}{-g_m G_{gen} \left(\frac{R_{gen} C_L C_{gs}}{g_m} s^2 + \frac{(C_L + C_{gs})}{g_m} s + 1\right)}$$

$$\frac{V_{out}}{V_{gen}} = \frac{\left(\frac{C_{gs}}{g_m} s + 1\right)}{\left(\frac{R_{gen} C_L C_{gs}}{g_m} s^2 + \frac{C_L + C_{gs}}{g_m} s + 1\right)}$$

e.) (2nd e)

for Denominator $as^2 + bs + c$

$$a = \frac{R_{gen} C_L C_{gs}}{g_m} = \frac{1M\Omega \cdot 1pF \cdot 1pF}{1mS} = 1\times10^{-15}\ s^2$$

$$b = \frac{C_L + C_{gs}}{g_m} = \frac{2\times10^{-12} F}{1mS} = 2\times10^{-9}\ s$$

$$c = 1$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2\times10^{-9}s \pm \sqrt{(2\times10^{-9}s)^2 - 4(1\times10^{-15}s^2)(1)}}{2(1\times10^{-15}s^2)}$$

$$s_{p1,2} = \underbrace{-1\times10^6}_{-\zeta\omega_n} \pm \underbrace{3.1\times10^7}_{\omega_p} j \ \left(\frac{rad}{s}\right)$$

$$\omega_n = \sqrt{(1\times10^6)^2 + (3.1\times10^7)^2} = 3.1\times10^7 \frac{rad}{s}$$

for numerator

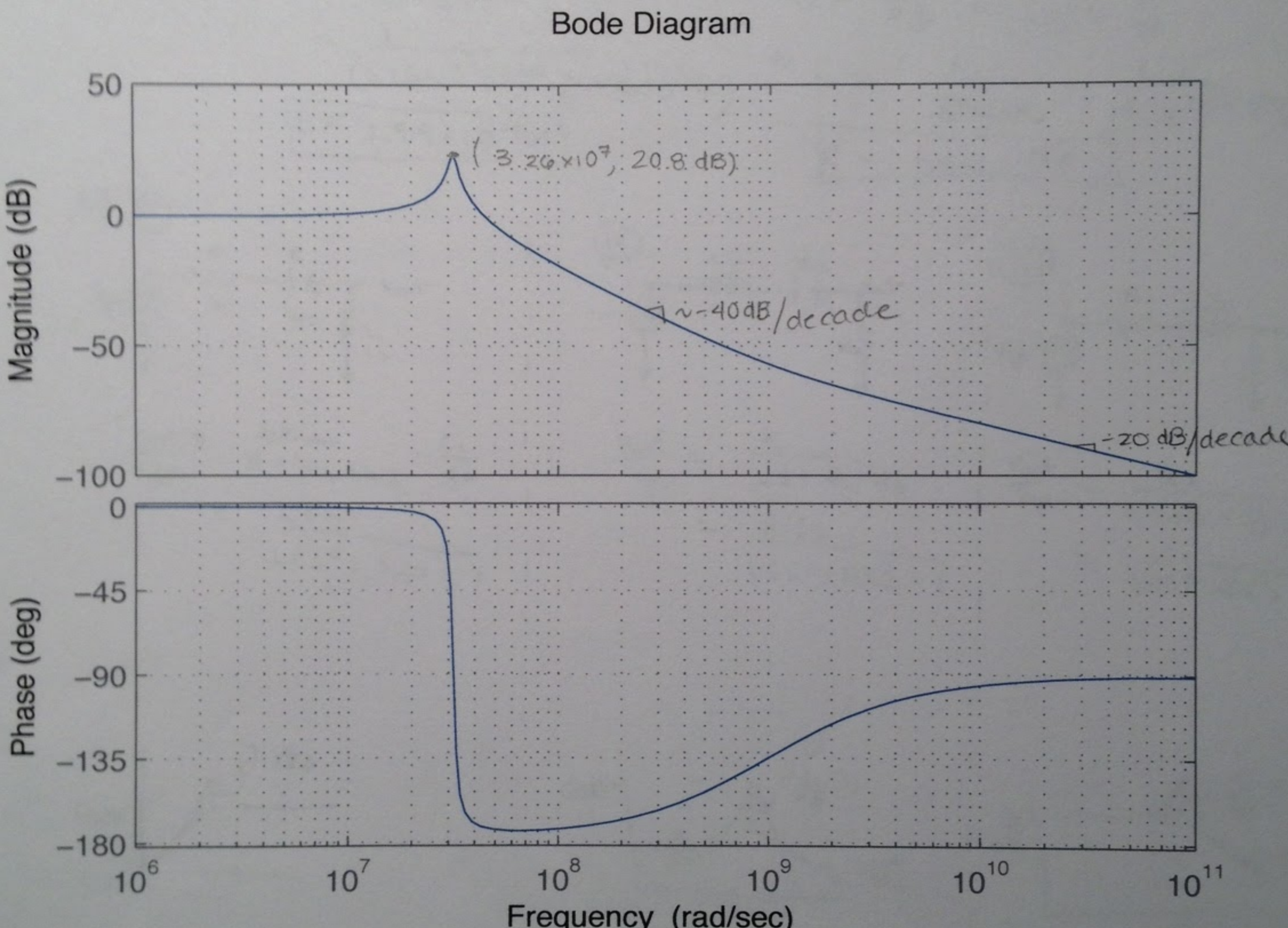
$$s_z = -\frac{g_m}{C_{gs}} = -\frac{1mS}{1pF} = -1\times10^9\ rad/s$$

$$\zeta\omega_n = 1\times10^6$$

$$\zeta = \frac{1\times10^6}{\omega_n} = \frac{1\times10^6}{3.1\times10^7}$$

$$\zeta = 3.2\times10^{-2}$$

1. e) (2nd e) (cont'd)



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2.) a) Series RLC → denominator = $LCs^2 + RCs + 1$

$\omega_n = 2\pi f = 1\,\text{GHz} \cdot 2\pi = 6.28\times10^9\ \text{rad/s}$ form: $\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1$

$$\frac{1}{\omega_n^2} = LC$$

$$\Downarrow$$

$$L = \frac{1}{\omega_n^2 C}$$

$$= \frac{1}{\left(6.28\times10^9\frac{\text{rad}}{\text{s}}\right)^2\cdot\left(1.59\times10^{-14}F\right)}$$

$$\boxed{L = 1.59\times10^{-6}H}$$

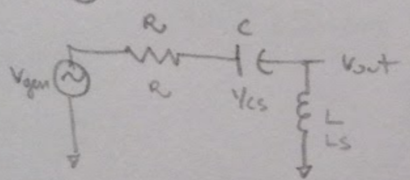
$$\frac{2\zeta}{\omega_n} = Rc \Rightarrow c = \frac{2\zeta}{\omega_n R}$$

$$Q = \frac{1}{2\zeta} \Rightarrow 2\zeta = \frac{1}{Q}$$

$$\Rightarrow C = \frac{1}{Q\omega_n R} = \frac{1}{10\cdot 6.28\times10^9\frac{\text{rad}}{\text{s}}\cdot 1\,k\Omega}$$

$$\boxed{C = 1.59\times10^{-14}F}$$

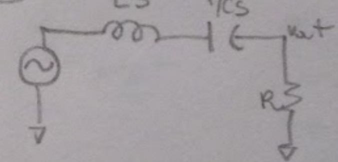
b.) i



$$\frac{V_{out}}{V_{gen}} = \frac{Ls}{R + 1/cs + Ls}\cdot\frac{Cs}{Cs}$$

$$= \frac{LCs^2}{LCs^2 + RCs + 1}$$

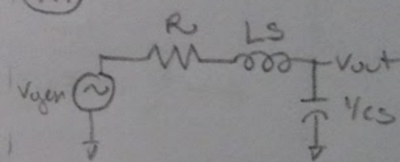
ii



$$\frac{V_{out}}{V_{gen}} = \frac{R}{LS + R + 1/cs}$$

$$= \frac{RCS}{LCS^2 + RCS + 1}$$

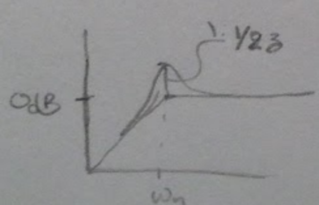
iii

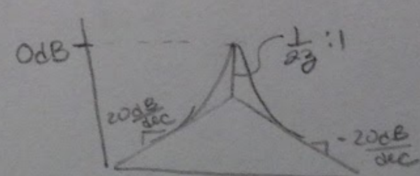


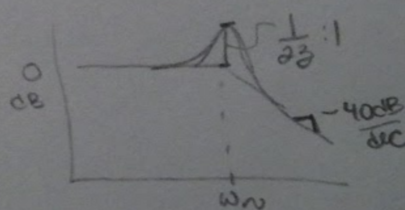
$$\frac{V_{out}}{V_{gen}} = \frac{1/cs}{LS + R + 1/cs}$$

$$= \frac{1}{LCS^2 + RCS + 1}$$

c.)







d.) $V_{in} = 1mV \cdot 1pS \cdot \delta(t) = V_0 \tau \delta(t)$

$V_{in}(s) = 1mV \cdot 1pS = V_0 \tau$

$2z/\omega_n = RC$

$\omega_n^2 = \frac{1}{LC}$

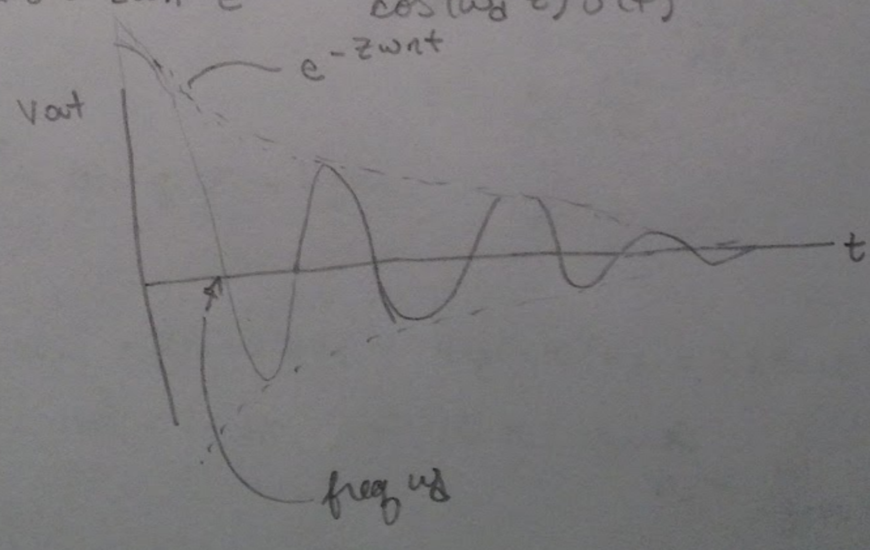
$$V_{out} = \frac{RCs \cdot V_0 \tau}{(LCs^2 + RCs + 1)} = \frac{V_0 \tau \frac{2z}{\omega_n} \cdot s}{s^2/\omega_n^2 + \frac{2z}{\omega_n} \cdot s + 1} \cdot \frac{\omega_n^2}{\omega_n^2} = \frac{V_0 \tau \, 2z\omega_n \cdot s}{s^2 + 2z\omega_n s + \omega_n^2}$$

$$= (V_0 \tau \, 2z\omega_n) \frac{s}{(s + z\omega_n)^2 + \omega_d^2}$$ where $\omega_d = \omega_n \sqrt{1 - z^2}$

$$= V_0 \tau \, 2z\omega_n \left[\frac{(s + z\omega_n - z\omega_n)}{(s + z\omega_n)^2 + \omega_d^2} \right]$$

$$V_0 \tau \, 2z\omega_n \left[\frac{(s + z\omega_n)}{(s + z\omega_n)^2 + \omega_d^2} - \underbrace{\underbrace{\frac{z\omega_n}{\omega_d}}_{\sim .05} \frac{\omega_d}{(s + z\omega_n)^2 + \omega_d^2}}_{\text{neglect this term}} \right]$$

$$V_{out}(t) \sim 2V_0 \tau \, z\omega_n \, e^{-z\omega_n t} \cos(\omega_d t) \, u(t)$$



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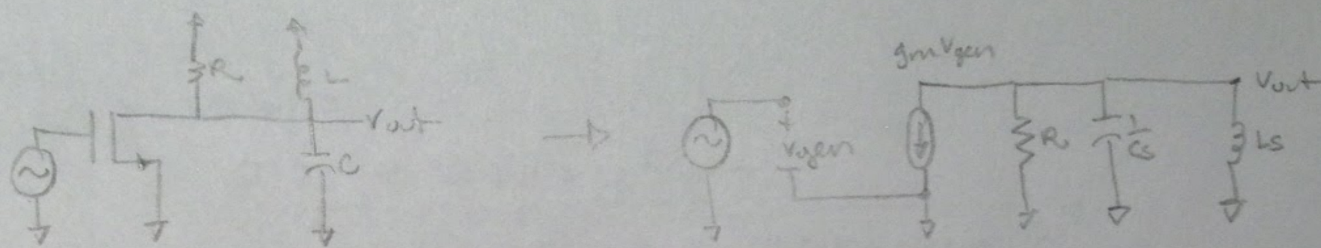
) a.)



$$g_m V_{gen} + V_{out}\left(\frac{1}{Ls} + Cs + G\right) = 0$$

$$\frac{V_{out}}{V_{gen}} = \frac{-g_m}{\frac{1}{Ls} + Cs + G} \cdot \frac{sL}{sL} = \frac{-g_m L s}{LCs^2 + GLs + 1}$$

standard form $= A \dfrac{\left(\frac{2z}{\omega_n}\right) s}{\frac{s^2}{\omega_n^2} + \frac{2z}{\omega_n} s + 1}$

$$\frac{1}{\omega_n^2} = LC \qquad \frac{2z}{\omega_n} = GL \qquad A = -\frac{g_m}{G} = -g_m R$$

$$\omega_n = \frac{1}{\sqrt{LC}} \qquad z = \frac{GL\omega_n}{2}$$

$$z = \frac{GL}{2\sqrt{LC}} = \frac{1}{2R}\sqrt{\frac{L^2}{LC}}$$

$$z = \frac{1}{2R}\sqrt{\frac{L}{C}}$$

$$\frac{A2z}{\omega_n} = GL\,g_m R = Lg_m$$

• we want $\omega_n = 2\pi \cdot 2\,\text{GHz} = 4 \cdot pi \cdot 10^9\,\text{Hz} = 1.25\times10^{10}$ rad/s

@ ω_n $|H| = 20\,\text{dB} \Rightarrow 10 = -g_m R$

$$R = \frac{10}{g_m} = \frac{10}{20\,\text{mS}} = \boxed{500\Omega}$$

17dB @ ω_L

$$\omega_L = \omega_n\left[\sqrt{1 - z^2} - z\right]$$

@ $\omega_m = 2\pi \cdot 1.5\,\text{GHz} = 9.4\times10^9$ rad/s we want

$|H| = -20\,\text{dB} \Rightarrow .1$

$$|H| = \frac{A \cdot 2z \cdot \omega_n \cdot \omega}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2z\omega_n\omega)^2}} = \frac{A2z\,f_n f}{\sqrt{(f_n^2 - f^2)^2 + (2z f_n f)^2}}$$

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3 a.) continued

$|H| = .1 @ f = 1.5\ GHz$

$$|H| = .1 = \frac{10 \cdot 2 \cdot z \cdot 2\ \cancel{GHz} \cdot 1.5\ \cancel{GHz}}{\sqrt{(2\ \cancel{GHz}^2 - 1.5\ \cancel{GHz}^2)^2 + (z \cdot 2 \cdot 1.5\ \cancel{GHz} \cdot 2\ \cancel{GHz})^2}}$$

$$(.1)^2 = \left(\frac{60z}{\sqrt{3.0625 + 36z^2}}\right)^2$$

$$.01 = \frac{3600z^2}{3.0625 + 36z^2}$$

$$(.01)(3.0625 + 36z^2) = 3600z^2$$

$$.030625 + .36z^2 = 3600z^2$$

$$.030625 \approx 3600z^2$$

$$0.0029 \sim z$$

$$L = \frac{R \cdot 2z}{\omega_n} = \frac{500\Omega \cdot 2 \cdot .0029}{1.25 \times 10^{10}\ rad/s} = \boxed{2.32 \times 10^{-10}\ H}$$

$$C = \frac{1}{\omega_n^2 L} = \frac{1}{(1.25 \times 10^{10}\ \frac{rad}{s})^2 (2.32 \times 10^{-10} H)} = \boxed{2.76 \times 10^{-11}\ F}$$

b.) Need to find gain in dB @ 1.8 GHz and 2.2 GHz

$$|H @ 1.8\ GHz| = \frac{10 \cdot 2 \cdot .0029 \cdot 2\ GHz \cdot 1.8\ GHz}{\sqrt{(2^2 - 1.8^2)^2 + (.0029 \cdot 2 \cdot 1.8 \cdot 2)^2}} = 0.275 \rightarrow -11.2\ dB$$

$$|H @ 2.2\ GHz| = \frac{10 \cdot 2 \cdot .0029 \cdot 2 \cdot 2.2}{\sqrt{(2^2 - 2.2^2)^2 + (.0029 \cdot 2 \cdot 2.2 \cdot 2)^2}} = 0.304 \rightarrow -10.3\ dB$$

max dB change is $20dB - -11.2dB = \boxed{31.2dB}$

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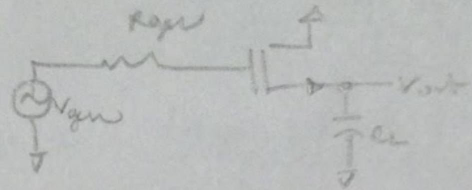
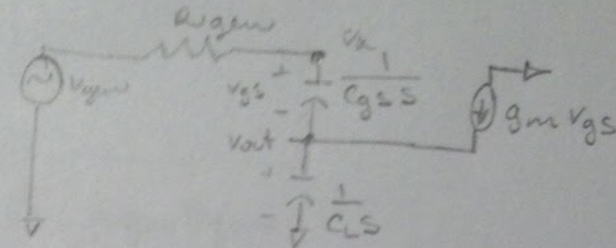
4.)





$$V_x = V_{gs} + V_{out}$$

@ node V_x

$$(V_{gen} - V_x)\, G_{gen} = (V_x - V_{out})\, C_{gs} S$$

$$(V_{gen} - V_{gs} - V_{out})\, G_{gen} = (V_{gs})\, C_{gs} S$$

$$V_{gen} G_{gen} = V_{gs}(G_{gen} + C_{gs} S) + V_{out} G_{gen}$$

@ node V_{out}

$$g_m V_{gs} + (V_x - V_{out})\, C_{gs} S = V_{out} C_L S$$

$$g_m V_{gs} + V_{gs} C_{gs} S - V_{out} C_L S = 0$$

$$V_{gs}(g_m + C_{gs} S) + V_{out}(-C_L S) = 0$$

$$\begin{bmatrix} G_{gen} + C_{gs} S & G_{gen} \\ g_m + C_{gs} S & -C_L S \end{bmatrix} \begin{bmatrix} V_{gs} \\ V_{out} \end{bmatrix} = \begin{bmatrix} V_{gen} G_{gen} \\ 0 \end{bmatrix}$$

$$D = \begin{vmatrix} G_{gen} + C_{gs} S & G_{gen} \\ g_m + C_{gs} S & -C_L S \end{vmatrix} = -C_L C_{gs} S^2 - C_L G_{gen} S - C_{gs} G_{gen} S - g_m G_{gen}$$

$$= -C_L C_{gs} S^2 + G_{gen}(-C_{gs} - C_L) S - g_m G_{gen}$$

$$N = \begin{vmatrix} G_{gen} + C_{gs} S & V_{gen} G_{gen} \\ g_m + C_{gs} S & 0 \end{vmatrix} = -V_{gen} G_{gen}(C_{gs} S + g_m)$$

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4.) a)
$$\frac{V_{out}}{V_{gen}} = \frac{G_{gen}(C_{gs}s + g_m)}{+C_L C_{gs} s^2 + G_{gen}(C_{gs}+C_L)s + g_m G_{gen}}$$

$$= \frac{1/g_m \cdot (C_{gs}s + g_m)}{1/g_m (+C_L C_{gs} R_{gen} s^2 + (C_{gs}+C_L)s + g_m)}$$

$$\boxed{\frac{V_{out}(s)}{V_{gen}(s)} = \frac{\left(\frac{C_{gs}}{g_m}s + 1\right)}{\left(+\frac{C_L C_{gs} R_{gen}}{g_m}s^2 + \left(\frac{C_{gs}+C_L}{g_m}\right)s + 1\right)}} = \frac{(1\times10^{-10}s + 1)}{(1\times10^{-18}s^2 + 1.1\times10^{-9}s + 1)}$$

$$\frac{1}{\omega_n^2} = \frac{C_L C_{gs} R_{gen}}{g_m} = 1\times10^{-18}$$

$$\frac{2\zeta}{\omega_n} = \frac{C_{gs}+C_L}{g_m} = 1.1\times10^{-9}$$

b.)
$$H(s) \sim \frac{1}{(10^{-18}s^2 + 1.1\times10^{-9}s + 1)}$$

$$s = \frac{-1.1\times10^{-9} \pm \sqrt{(1.1\times10^{-9})^2 - 4(10^{-18})}}{2\times10^{-18}}$$

$$s = -5.5\times10^{8} \pm 8.35\times10^{8}j$$

$$V_{gen}(t) = 1\,mV\,u(t)$$

$$V_{gen}(s) = \frac{1\,mV}{s}$$

$$V_{out}(s) = H(s)\cdot V_{gen}(s)$$

$$= \frac{1\,mV}{s(10^{-18}s^2 + 1.1\times10^{-9}s + 1)} = \frac{1\,mV}{s(s+5.5\times10^{8} - 8.35\times10^{8}j)(s+5.5\times10^{8} + 8.35\times10^{8}j)}$$

$$= \frac{A}{s} + \frac{B}{(s+5.5\times10^{8} - 8.35\times10^{8}j)} + \frac{B^*}{(s+5.5\times10^{8} + 8.35\times10^{8}j)}$$

$$s=0 \Rightarrow A = 1\,mV = .001$$

$$s = -5.5\times10^{8} + 8.35\times10^{8}j$$

$$\Rightarrow 1\,mV = B(-5.5\times10^{8} + 8.35\times10^{8}j)(2\cdot 8.35\times10^{8}j)$$

$$B = \frac{1\,mV}{(-9.18\times10^{17}j - 1.39\times10^{18})}$$

$$= -5.01\times10^{-22} + 3.31\times10^{-22}j$$

$$B^* = -5.01\times10^{-22} - 3.31\times10^{-22}j$$

4b contd

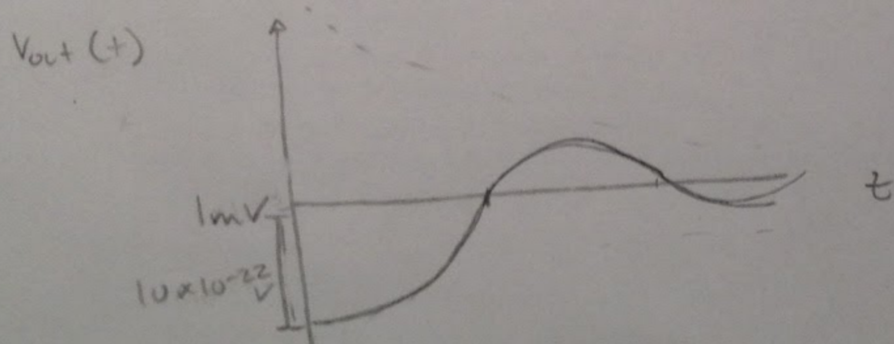
$$V_{out}(s) = \frac{.001\,V}{s} + \frac{-5.01\times10^{-22}+3.31\times10^{-22}j}{(s+5.5\times10^{8}-8.35\times10^{8}j)} + \frac{-5.01\times10^{-22}-3.31\times10^{-22}j}{(s+5.5\times10^{8}+8.35\times10^{8}j)}$$

$$V_{out}(t) = .001 + (-5.01\times10^{-22}+j3.31\times10^{-22})\times e^{(-5.5\times10^{8}+8.35\times10^{8}j)t} + (-5.01\times10^{-22}-j3.31\times10^{-22})\times e^{(-5.5\times10^{8}-8.35\times10^{8}j)t}$$

$$= .001 + (-5.01\times10^{-22})(e^{-5.5\times10^{8}t})(e^{8.35\times10^{8}jt}+e^{-j8.35\times10^{8}t}) + j\,3.31\times10^{-22}(e^{-5.5\times10^{8}t})(e^{j8.35\times10^{8}t}-e^{-j8.35\times10^{8}t})$$

$$= .001 + (-5.01\times10^{-22})e^{-5.5\times10^{8}t}\cdot 2\cos(8.35\times10^{8}t) - (3.31\times10^{-22})e^{-5.5\times10^{8}t}\cdot 2\sin(8.35\times10^{8}t)$$

$$V_{out}(t) = \left[.001 + e^{-5.5\times10^{8}t}\left[-1.02\times10^{-21}\cos(8.35\times10^{8}t) - 6.62\times10^{-22}\sin(8.35\times10^{8}t)\right]\right]u(t)\ V$$



• no overshoot

10-90% rise time $= 2.2\tau = 2.2\times\left(\frac{1}{5.5\times10^{8}}\right) = \boxed{4\times10^{-9}s}$