ECE 2C Mid-Term Exam

May 9, 2013

Do not open exam until instructed to.

Closed book: 2 pages personal notes permitted

There are xx problems on this exam, and you have 75 minutes.

Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING and approximately Justifying them.***

(1) Answers without units are wrong.
(2)Plots without correctly labelled and dimensioned axes are wrong.

Name: solution - exam "a"

Perm #: ________________________

Problem	Points Received	Points Possible
1a		5
1b		5
1c		10
2a		5
2b		5
3a		5
3b		10
3c		10
4a		10
4b		10
~~5a~~		~~10~~
~~5b~~		~~10~~
total		~~100~~ 75

Problem 1, 15 points

Part a, 5 points.

Q1 is a mobility-limited FET, ie.

$I_d = (\mu C_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$ where

$(\mu C_{ox} W_g / 2L_g)$=1 mA/V^2, $\lambda = 0.0\ \mathrm{V}^{-1}$, and V_{th}=0.3 V.

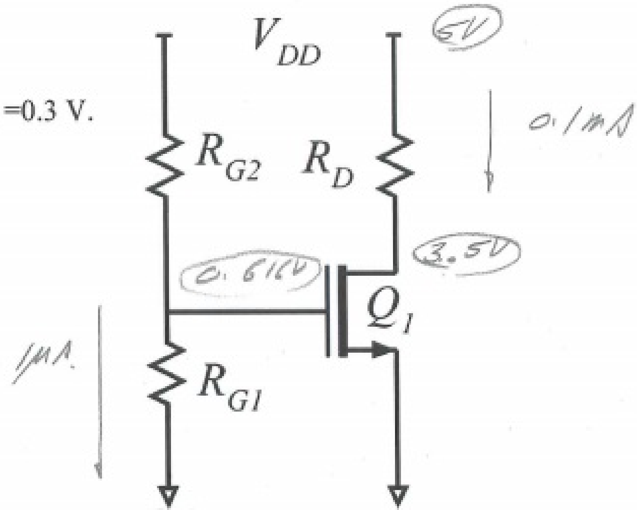
V_{DD}=5 Volts

The drain is biased at +3.5 Volts.

The drain current is 0.1 mA.

The DC current in R_{G1} is 1 μA

Find R_{G1}, R_{G2}, R_D, and the DC gate voltage.



R_{G1}= 616kΩ R_{G2}= 4.58MΩ R_D= 15kΩ

V_{G1}= 0.616V.

1pt $\left[\ R_D = (5V - 3.5V)/0.1mA = 1.5V/0.1mA = 15k\Omega \right.$

2pt $\left[\ 0.1\,mA = 1\,\frac{mA}{V^2}(V_{gs} - 0.3V)^2 \rightarrow 0.1\,V^2 = (V_{gs} - 0.3V)^2 \right.$

$\rightarrow V_{gs} = 0.616V$

1pt $\left[\ R_{g1} = \frac{0.616V}{1\mu A} = 616k\Omega \right.$

1pt, $\left[\ R_{g2} = \frac{5V - 0.616V}{1mA} = \frac{4.584V}{1\mu A} = 4.58M\Omega \right.$

Part b, 5 points

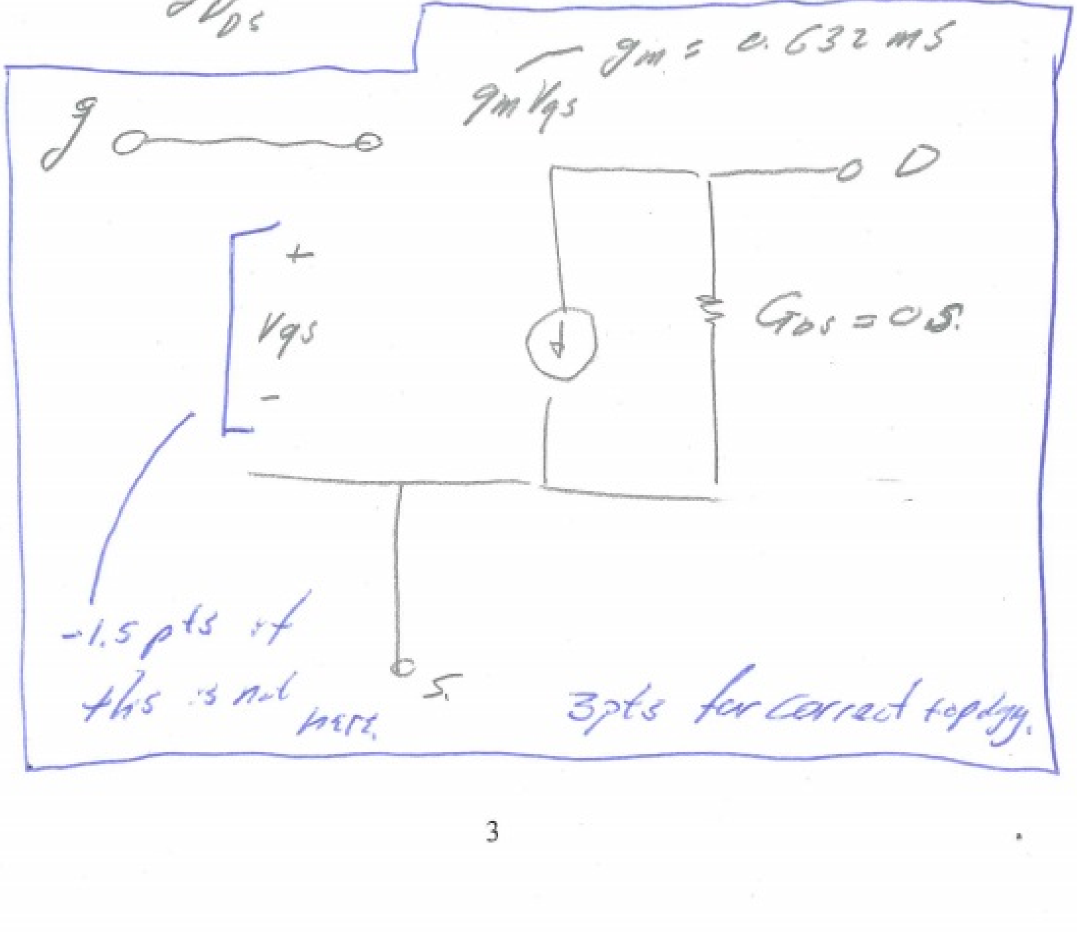
Using the FET parameters and DC bias conditions of part (a), ***draw a small-signal model of the FET and give numerical values of all small-signal parameters.***

FET operating @ $V_{gs} = 0.616V$, $V_{DS} = 3.5$

$$I_d = \frac{1mA}{V^2}\cdot(V_{gs}-\overset{0.3V}{V_{t}})^2(1+\overset{0}{\lambda}V_{DS})$$

2 $$g_m \triangleq \frac{\partial I_D}{\partial V_{gs}} = \frac{2mA}{V^2}\cdot(\overset{0.616}{V_{gs}}-0.3V) = \underline{0.632\ mS}$$

1 $$G_{DS} = \frac{\partial I_D}{\partial V_{DS}} = 0 \quad \text{because } \lambda = 0$$



-1.5 pts if this is not here.

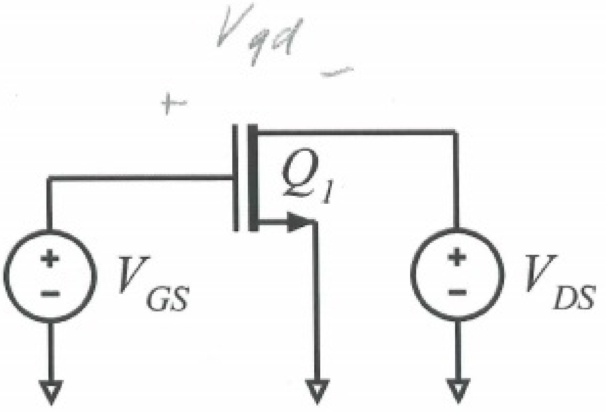
3pts for correct topology.

Part c, 10 points.

When FET gate lengths become *very* short, the neither the mobility-limited nor the velocity-limited models are correct. Instead, we have ***degenerate/ballistic*** characteristics, which for ***low Vds*** gives:

$I_d = K(V_{gs} - V_{th})^{3/2} - K(V_{gd} - V_{th})^{3/2}$

For this FET, K=1 $\text{mA/V}^{3/2}$, and V_{th}=0.2 V.



In the circuit shown, the gate is biased at +0.8Volts, and the drain is biased at +0.2 Volts. ***Draw a small-signal model of the FET*** labelling all elements and all control voltages, ***and give numerical values of all small-signal parameters***.

$$I_d = k(V_{gs} - V_{th})^{3/2} - k(V_{gd} - V_{th})^{3/2}$$

1 key: $V_{gd} = V_{gs} - V_{DS}$ (important!)

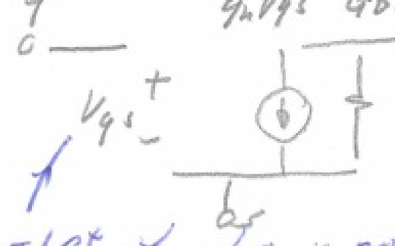
1 $$I_d = k(V_{gs} - V_{th})^{3/2} - k(V_{gs} - V_{DS} - V_{th})^{3/2}$$

2 $$g_m \triangleq \frac{\partial I_d}{\partial V_{gs}} = \frac{3k}{2}(V_{gs} - V_{th})^{1/2} - \frac{3k}{2}(V_{gs} - V_{DS} - V_{th})^{1/2}$$

1 $$= \frac{3}{2}\frac{mA}{V^{3/2}}(0.8V - 0.2V)^{1/2} - \frac{3}{2}\frac{mA}{V^{3/2}}(0.8V - 0.4V)^{1/2} = \cdot 0.213\ \underline{\underline{mS}}$$

2 $$G_{ds} \triangleq \frac{\partial I_D}{\partial V_{DS}} = (+)\frac{3k}{2}(V_{gs} - V_{ds} - V_{th})^{1/2} = \frac{3mA}{2V^{3/2}}(0.8V - 0.4V)^{1/2} = 0.95mS$$

①



2

−1pt if this is not here.

Problem 2, 15 points

Q1 is a mobility-limited FET, ie.

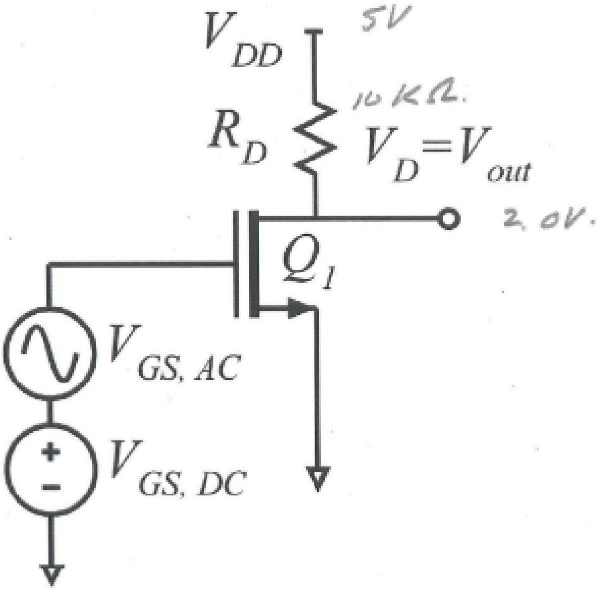
$I_d = (\mu C_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$

where $(\mu C_{ox} W_g / 2L_g)$=1 mA/V^2,

$\lambda = 0.1\,\mathrm{V}^{-1}$, and V_{th}=0.3 V.

V_{DD}=5 Volts

R_D=10 $k\Omega$



Part a, 5 points

We wish to have a DC drain voltage of 2.0 Volts.
What DC gate-source voltage does this require ?

$V_{GS,DC}$ = 0.8V.

2 $I_D = \frac{5V - 2V}{10\,k\Omega} = \frac{3V}{10\,k\Omega} = 0.3\,mA$

3 $I_d = 0.3\,mA = \frac{1\,mA}{V^2}(V_{gs} - 0.3V)^2\left(1 + \frac{\overset{2V}{\cancel{5}}\;V_{ds}}{10V}\right)$

$0.25\,V^2 = (V_{gs} - 0.3V)^2 \longrightarrow V_{gs} = 0.8V$

Part b, 5 points

$V_{GS,AC}(t) = 1\,\mathrm{mV} \cdot \cos(2\pi \cdot 1\mathrm{kHz} \cdot t)$

What is the AC drain voltage ?

$V_{D,AC}(t) =$ ________________________

we need ss model (!)

ok to drop.

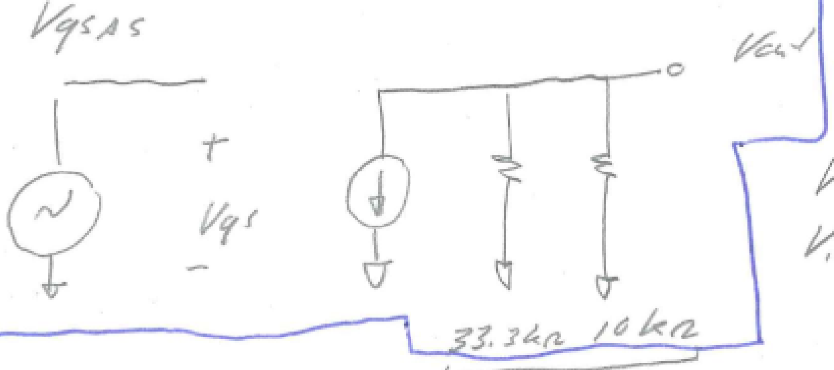
1/4 $g_m \cong 2\,(\mu C_{ox} W_g/(2L_g))(V_{gs} - V_{th})(1 + \lambda V_{ds})$

1/4 $= 2\frac{mA}{V^2}(0.8V - 0.3V) = \underline{\underline{1mS}}$

1/4 $G_{ds} \triangleq \frac{\partial I_D}{\partial V_{DS}} = \frac{\lambda I_D}{1+\lambda V_{DS}} \cong \lambda I_D = \underbrace{\frac{0.3mA}{10V} = 30\mu S = \frac{1}{33.3k\Omega}}_{1/4}$

2

small signal model:



$\frac{V_{out}}{V_i} = -g_m R_{eq}$

$= -7.69$

1/2

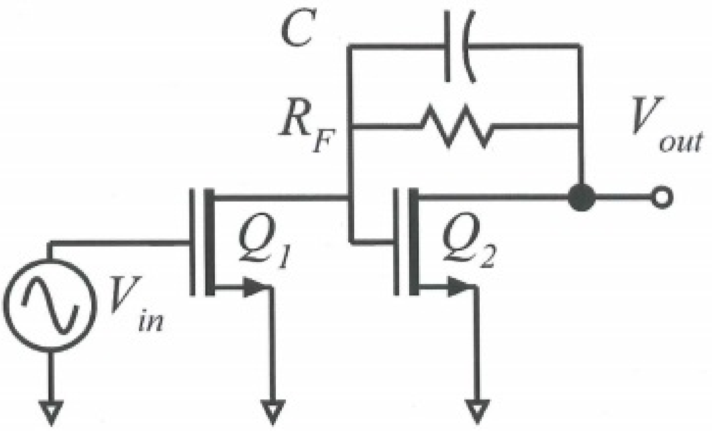
$R_{eq} = 7.69k\Omega$ 1/2

1 $V_{out}(t) = -7.69mV \cdot \cos(2\pi \cdot 1kHz \cdot t)$

Problem 3, 25 points

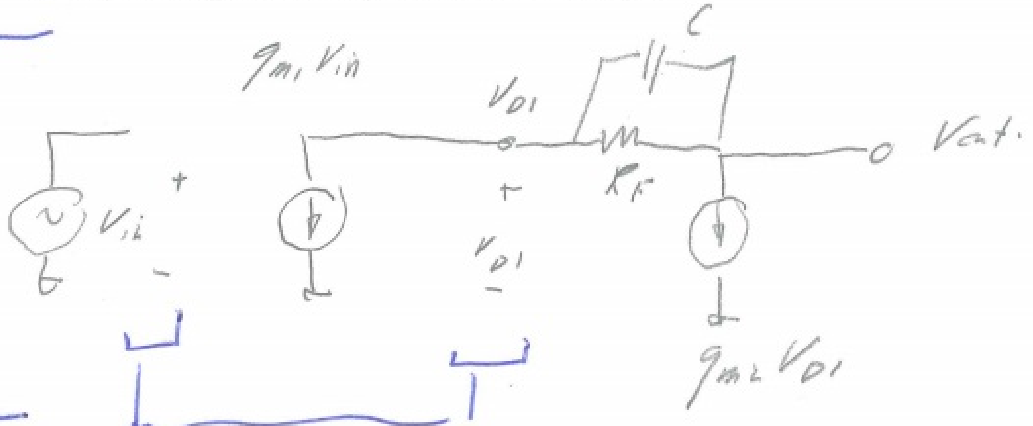
You will be working with the circuit at right. Ignore DC bias. You don't need it.

The transistors Q1 and Q2 have transonductances g_{m1} and g_{m2}, ***both nonzero,*** but $G_{ds1} = G_{ds2}$ **=0 mS.** The transistors have no gate-source or gate-drain capacitances.



Part a, 5 points

replacing the transistors with their small-signal models, draw a small-signal equivalent circuit, labelling all elements and all control voltages.



topology must be correct

component labels must be present

these must be labelled.

Part b, 10 points

Compute $V_{out}(s)/V_{in}(s)$: The answer must be in one of these two standard forms:

$$\frac{V_{out}(s)}{V_{in}(s)} = \left.\frac{V_{out}}{V_{in}}\right|_{\substack{\text{low-frequency}\\ \text{value}}} \times \frac{1+b_1 s+b_2 s^2+\ldots}{1+a_1 s+a_2 s^2+\ldots} \quad \text{or}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \left.\frac{V_{out}}{V_{in}}\right|_{\substack{\text{low-frequency}\\ \text{value}}} \times \frac{(1-s/s_{z1})(1-s/s_{z2})\ldots}{(1-s/s_{p1})(1-s/s_{p2})\ldots}$$

$V_{out}(s)/V_{in}(s)=$ ______________________________

3 $\boxed{\Sigma I = 0 \text{ @ } V_{D1}}$ $\boxed{g_{m1}V_{in} + V_{D1}(sC + G_F) + V_{out}(-sC - G_F) = 0}$

$\rightarrow V_{D1}(sC + G_F) + V_{out}(-sC - G_F) = -g_m V_{in}$

3 $\boxed{\Sigma I = 0 \text{ @ } V_{out}}$

$\boxed{V_{out}(sC + G_f) + V_{D1}(g_{m2} - sC - G_F) = 0}$

$$\begin{bmatrix} sC + G_F & -sC - G_F \\ g_{m2} - sC - G_F & sC + G_F \end{bmatrix} \begin{bmatrix} V_{D1} \\ V_{out} \end{bmatrix} = \begin{bmatrix} -g_m V_i \\ 0 \end{bmatrix}$$

$$D(s) = \begin{vmatrix} sC + G_F & -sC - G_F \\ g_{m2} - sC - G_F & sC + G_F \end{vmatrix} = g_{m2}(sC + G_F)$$

$$N(s) = \begin{vmatrix} \text{[illegible]} & -g_{m1} V_{in} \\ g_{m2} - sC - G_F & 0 \end{vmatrix} = g_{m1}(g_{m2} - sC - G_F)$$

$$= \frac{g_{m1}(g_{m2} - sC - G_F)}{g_{m2}(sC + G_F)}$$

$$= \frac{g_{m1}(g_{m2} - G_F)}{g_{m2} G_F} \; \frac{(1 - sC/(g_{m2} - G_F))}{1 + sC/G_F}$$

$$\boxed{\frac{V_o(s)}{V_{gn}(s)} = \frac{g_{m1}}{g_{m2}}(g_{m2}R_F - 1) \; \frac{1 - sC(g_{m2} - G_F)^{-1}}{1 + sCR_F}}$$

②

Part c, 10 points

Now $g_{m1} = g_{m2} = 1$ ms, R_f=10 kΩ, C= 1 nF.

Give the frequencies, in Hz, of any and all poles and zeros. Clearly indicate if any lie in the right half of the S-plane

1st pole frequency=__15.9 kHz__, 2nd pole frequency=____________,
3rd pole frequency=____________

1st zero frequency=__143 kHz__, 2nd zero frequency=____________,
3rd zero frequency=____________

pole:

3 $\left[\tau = C \cdot R_F = 1nF \cdot 10k\Omega = 10\mu s \right.$

2 $\left[f_p = \frac{0.159}{10\mu s} = 15.9\ kHz \right.$

zero – in RHP.

3 $\left[\tau = \frac{C}{g_{m2} - G_f} = \frac{1nF}{1ms - 0.1ms} = 1.111\mu s \right.$

2 $\left[f_{zero} = \frac{0.159}{1.111\mu s} = 143\ kHz. \right.$

H0:

Problem 4, 20 points

We have a circuit for which

$$\frac{V_{out}(s)}{V_{in}(s)} = H(s) = H_{DC} \times \frac{1+s\tau_1}{(1+s\tau_2)(1+s\tau_3).}$$

where τ_1 = 1 ns, τ_2=10ns, τ_3=0.1 ns.

zero @ 160 MHz

poles: 16 MHz, 1.6 GHz

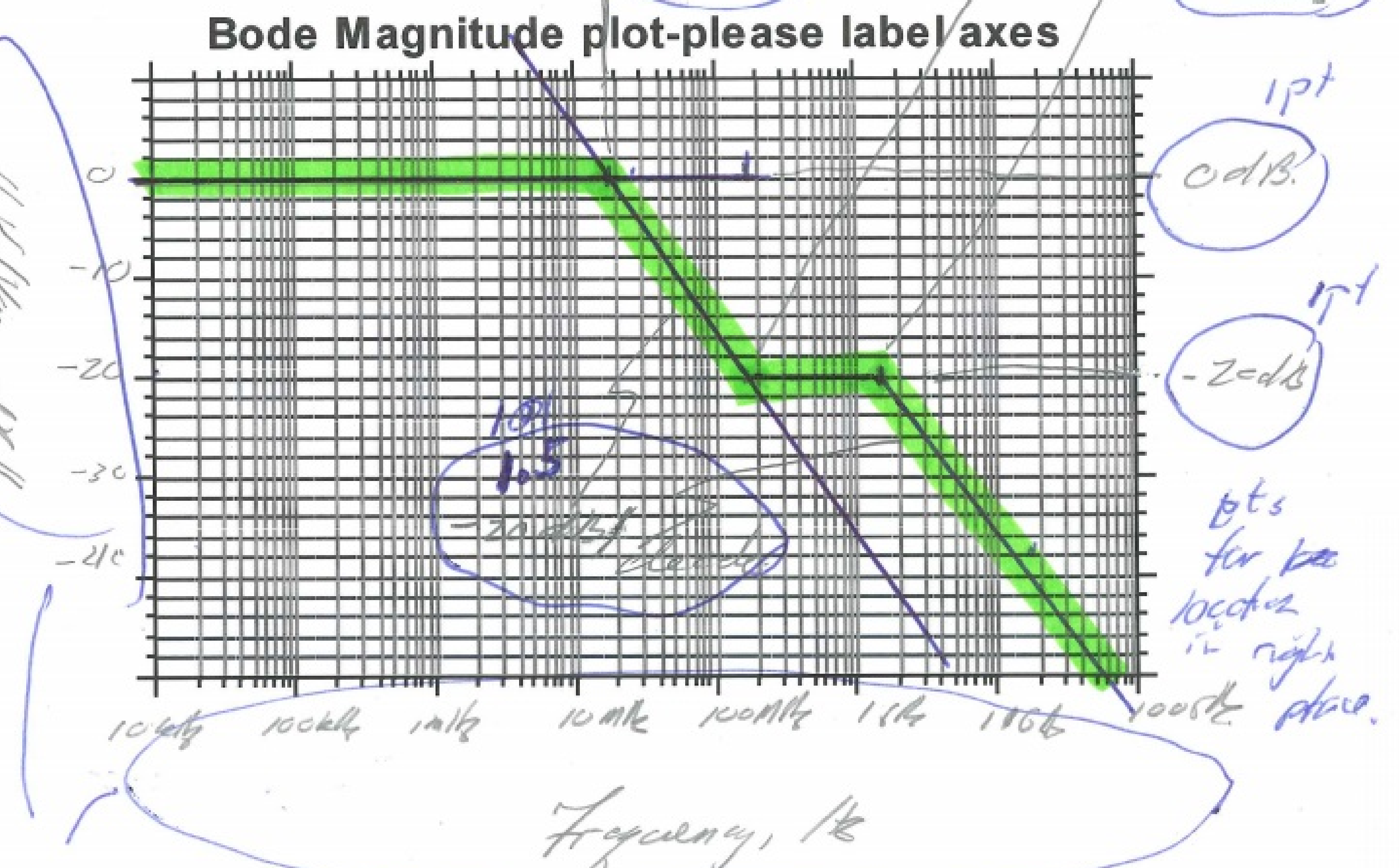
1/2 pt each

Part a, 10 points
Make an **accurate** asymptotic Bode plot of $\| H \|$, labelling and dimensioning axes and clearly labelling all slopes and all critical frequencies.

1pt fp1 = 16 MHz

1pt fz = 160 MHz

1pt fp2 = 1.6 GHz



1pt 0 dB.

1pt -20 dB

pts for be location in right place.

correct scales & labels: ~ 2pts.

a exam.

Part b, 10 points
Make an **accurate** root locus plot of H(s), labelling and dimensioning axes and clearly labelling all critical frequencies.

be sure to label axes with #s and units