ECE 2C Mid-term Exam B Solutions

Spring 2013

Problem 1, 15 points

Part a, 5 points.

Q1 is a mobility-limited FET, ie.

$I_d = (\mu C_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$ where

$(\mu C_{ox} W_g / 2L_g) = 1\ \text{mA}/\text{V}^2$, $\lambda = 0.0\ \text{V}^{-1}$, and $V_{th} = 0.3$ V.

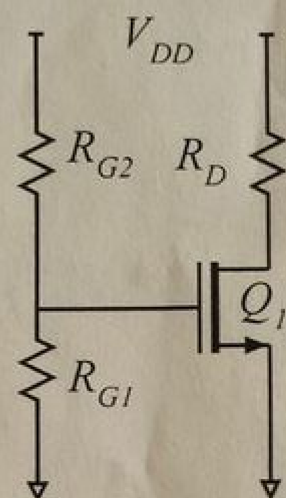
$V_{DD} = 5$ Volts

The drain is biased at +3.0 Volts.

The drain current is 0.25 mA.

The DC current in R_{G1} is 50 μA

Find R_{G1}, R_{G2}, R_D, and the DC gate voltage.



R_{G1}= 16k Ω R_{G2}= 84k Ω R_D= 8k Ω

V_{G1}= 0.8

$$R_D = \frac{5-3}{0.25\ mA} = 8k\Omega$$

$$0.25 \times 10^{-3} = 1 \times 10^{-3} (V_{gs} - 0.3)^2$$

$$0.25 = (V_{gs} - 0.3)^2$$

$$0.5 = V_{gs} - 0.3$$

$$V_{gs} = 0.8\ V$$

$$R_{g1} = \frac{0.8}{(50 \times 10^{-6})} = 16k\Omega$$

$$R_{g2} = \frac{5 - 0.8}{(50 \times 10^{-6})} = 84k\Omega$$

2

Part b, 5 points

Using the FET parameters and DC bias conditions of part (a), ***draw a small-signal model of the FET and give numerical values of all small-signal parameters.***

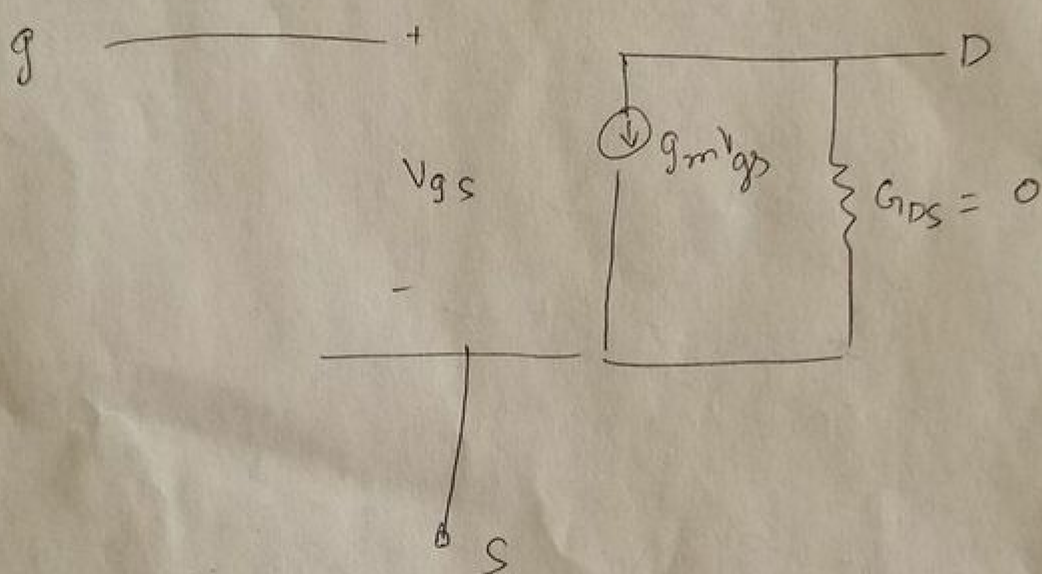
$V_g = 0.8\,V$

$g_m = \frac{\partial I_D}{\partial V_{gs}} = \frac{1\,mA}{V^2}(2)(V_{gs} - V_{th})$

$= \frac{2\,mA}{V^2}(0.5)$

$= 1\,mS$

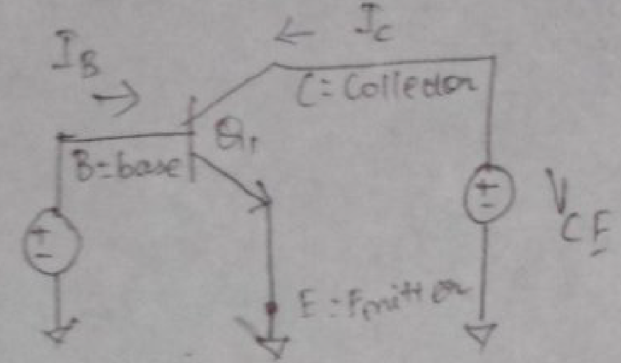
$G_{DS} = \frac{\partial I_D}{\partial V_{DS}} = 0$ since $\lambda = 0$



1)(c)

$$I_c = k(V_{be} - \phi)^2$$

$$I_B = \frac{I_c}{\beta} = \frac{k}{\beta}(V_{be} - \phi)^2$$



$k = 2\,mA/V^2$, $\phi = 0.9V$, $\beta = 100$

Small signal parameters are

$$G_{11} = \frac{\partial I_b}{\partial V_{be}} = \frac{2k}{\beta}(V_{be} - \phi)$$

$$= \frac{2 \times 2\frac{mA}{V^2}(1.1 - 0.9)}{100}$$

$$= (8 \times 10^{-3})\frac{mA}{V}$$

$$= 8\,\mu S = \frac{1}{(125 \times 10^3)\,\Omega}$$

$$G_{12} = \frac{\partial I_b}{\partial V_{ce}} = 0$$

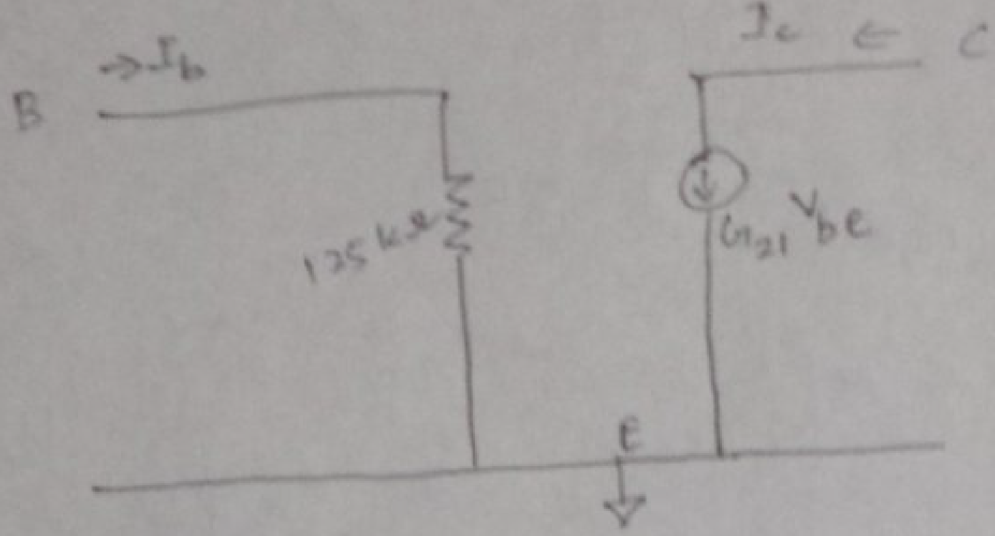
$$G_{21} = \frac{\partial I_c}{\partial V_{be}} = 2k(V_{be} - \phi)$$

$$= 2 \times 2 \times (1.1 - 0.9)\ \frac{mA}{V}$$

$$= 0.8\,mS$$

$$G_{22} = \frac{\partial I_c}{\partial V_{ce}} = 0$$

Small Signal model is



Problem 2, 15 points

Q1 is a mobility-limited FET, ie.

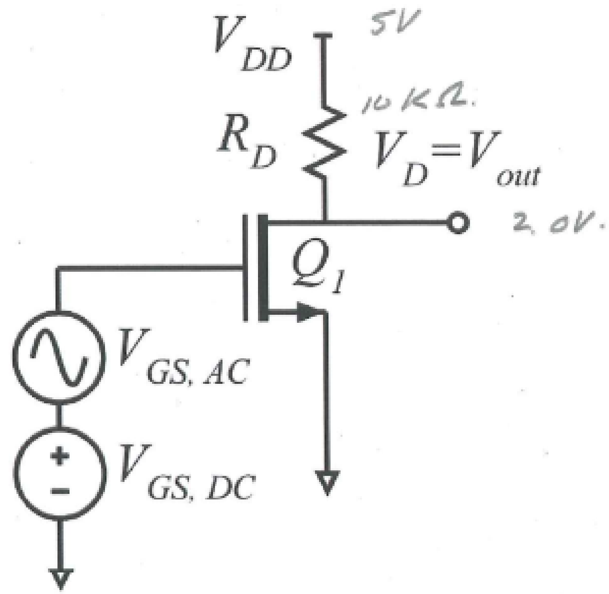
$I_d = (\mu C_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$

where $(\mu C_{ox} W_g / 2L_g)$=1 mA/V^2,

$\lambda = 0.1\,V^{-1}$, and V_{th}=0.3 V.

V_{DD}=5 Volts

R_D=10 $k\Omega$



Part a, 5 points

We wish to have a DC drain voltage of 2.0 Volts.
What DC gate-source voltage does this require ?

$V_{GS,DC}$ = 0.8 V.

2 $I_D = \frac{5V - 2V}{10\,k\Omega} = \frac{3V}{10\,k\Omega} = 0.3\,mA$

3 $I_d = 0.3\,mA = \frac{1\,mA}{V^2}(V_{gs} - 0.3V)^2 (1 + \frac{V_{ds}}{10V})$ (with V_{ds} = 2V)

$0.25\,V^2 = (V_{gs} - 0.3V)^2 \rightarrow V_{gs} = 0.8V$

Part b, 5 points

$V_{GS,AC}(t) = 1\,\text{mV} \cdot \cos(2\pi \cdot 1\text{kHz} \cdot t)$

What is the AC drain voltage ?

$V_{D,AC}(t) =$ ____________________

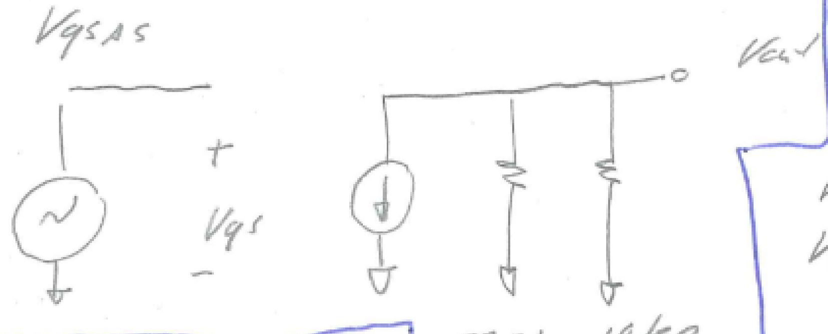
we need ss model (!) ok to drop.

1/4 $g_m \cong 2(\mu C_{ox} W_g/(2L_g))(V_{gs} - V_{th})(1 + \lambda V_{ds})$

1/4 $= 2\frac{mA}{V^2}(0.8V - 0.3V) = 1\,mS$

1/4 $G_{ds} \triangleq \frac{\partial I_D}{\partial V_{DS}} = \frac{\lambda I_D}{1+\lambda V_{DS}} \cong \lambda I_D = \frac{0.3mA}{10V} = 30\mu S = \frac{1}{33.3k\Omega}$ 1/4

2 small signal model:



$R_{eq} = 7.69\,k\Omega$ 1/2

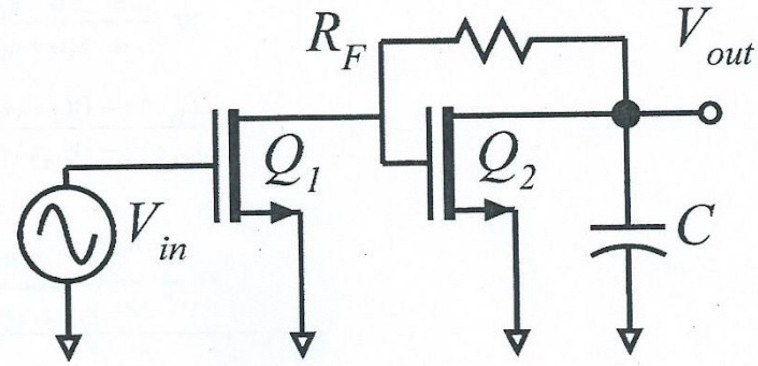
$\frac{V_{out}}{V_{in}} = -g_m R_{eq} = -7.69$ 1/2

1 $V_{out}(t) = -7.69\,mV \cdot \cos(2\pi \cdot 1kHz \cdot t)$

Problem 3, 25 points

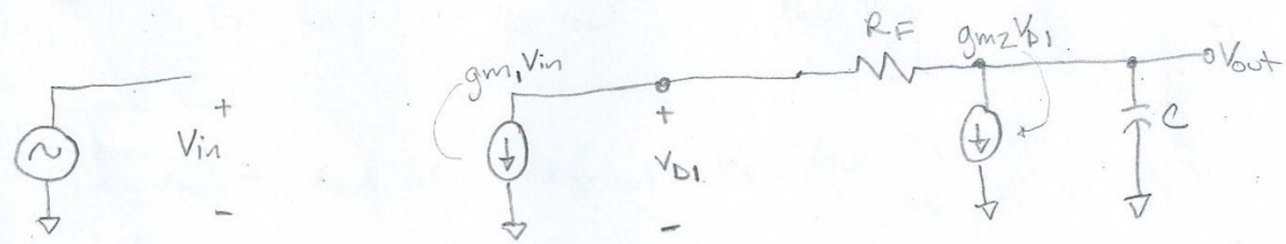
You will be working with the circuit at right. Ignore DC bias. You don't need it.

The transistors Q1 and Q2 have transonductances g_{m1} and g_{m2}, ***both nonzero,*** but $G_{ds1} = G_{ds2}$**=0 mS.** The transistors have no gate-source or gate-drain capacitances.



Part a, 5 points

replacing the transistors with their small-signal models, draw a small-signal equivalent circuit, labelling all elements and all control voltages.



Part b, 10 points

Compute $V_{out}(s)/V_{in}(s)$: The answer must be in one of these two standard forms:

$$\frac{V_{out}(s)}{V_{in}(s)} = \left.\frac{V_{out}}{V_{in}}\right|_{\substack{\text{low=frequency}\\ \text{value}}} \times \frac{1+b_1 s+b_2 s^2+\ldots}{1+a_1 s+a_2 s^2+\ldots} \quad \text{or}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \left.\frac{V_{out}}{V_{in}}\right|_{\substack{\text{low=frequency}\\ \text{value}}} \times \frac{(1-s/s_{z1})(1-s/s_{z2})\ldots}{(1-s/s_{p1})(1-s/s_{p2})\ldots}$$

$V_{out}(s)/V_{in}(s) =$ $\dfrac{g_{m1}(g_{m2}-G_F)}{G_F(g_{m2}+SC)}$

$\Sigma I = 0$ @ V_{D1}

$$g_{m1}V_{in} + (V_{D1}-V_{out})G_F = 0$$

$$\Rightarrow G_F V_{D1} - G_F V_{out} = -g_{m1}V_{in}$$

$\Sigma I = 0$ @ V_{out}

$$g_{m2}V_{D1} + V_{out}SC + (V_{out}-V_{D1})G_F = 0$$

$$\Rightarrow (g_{m2}-G_F)V_{D1} + (SC+G_F)V_{out} = 0$$

$$\begin{bmatrix} G_F & -G_F \\ g_{m2}-G_F & SC+G_F \end{bmatrix}\begin{bmatrix} V_{D1} \\ V_{out} \end{bmatrix} = \begin{bmatrix} -g_{m1}V_{in} \\ 0 \end{bmatrix}$$

Use Cramer's Rule:

$$D = \begin{vmatrix} G_F & -G_F \\ g_{m2}-G_F & SC+G_F \end{vmatrix} = G_F(SC+G_F) + G_F(g_{m2}-G_F)$$

$$= G_F(SC+g_{m2})$$

(solving for V_{out}, so replace 2nd column

$$N = \begin{bmatrix} G_F & -g_{m1}V_{in} \\ g_{m2}-G_F & 0 \end{bmatrix} = g_{m1}(g_{m2}-G_F)V_{in}$$

b

$$V_{out} = \frac{N(s)}{D(s)} = \frac{g_{m1}(g_{m2} - G_F)V_{in}}{G_F(g_{m2} + sC)}$$

$$\frac{V_{out}}{V_{in}} = H(s) = \frac{g_{m1}(g_{m2} - G_F)}{G_F(g_{m2} + sC)}$$

$$\boxed{\frac{V_{out}(s)}{V_{in}(s)} = \frac{g_{m1}(g_{m2} - G_F)}{G_F\, g_{m2}\left(1 + s\frac{C}{g_{m2}}\right)}}$$

Part c, 10 points

Now $g_{m1} = g_{m2} = 1$ ms, R_f=10 kΩ, C= 1 nF.

Give the frequencies, in Hz, of any and all poles and zeros. Clearly indicate if any lie in the right half of the S-plane

1st pole frequency=_159 KHz_, 2nd pole frequency=____________,
3rd pole frequency=____________

1st zero frequency=____________, 2nd zero frequency=____________,
3rd zero frequency=____________

pole

$$\tau = \frac{C}{g_{m2}} = \frac{1\,nF}{1\,mS} = 1\,\mu S$$

$$f_{pole} = \frac{1}{2\pi \cdot \tau} = \frac{0.159}{1\,\mu S} = 159\,kHz$$

Problem 4, 20 points

We have a circuit for which

$$\frac{V_{out}(s)}{V_{in}(s)} = H(s) = H_{DC} \times \frac{1+s\tau_1}{(1+s\tau_2)(1+s\tau_3)}. = \frac{1+s/10^7}{(1+s/10^8)(1+s/10^9)}$$

(H_{DC} = 1)

$\omega_{zero} = 10^7 = 2\pi f_z$ (rad/sec)

$\Rightarrow \omega_{pole1} = 10^8 = 2\pi f_{p1}$

$\omega_{pole2} = 10^9 = 2\pi f_{p2}$

where τ_1 = 100 ns, τ_2=10ns, τ_3=1 ns.

$\Rightarrow f_z = 1.6$ MHz

$f_{p1} = 16$ MHz

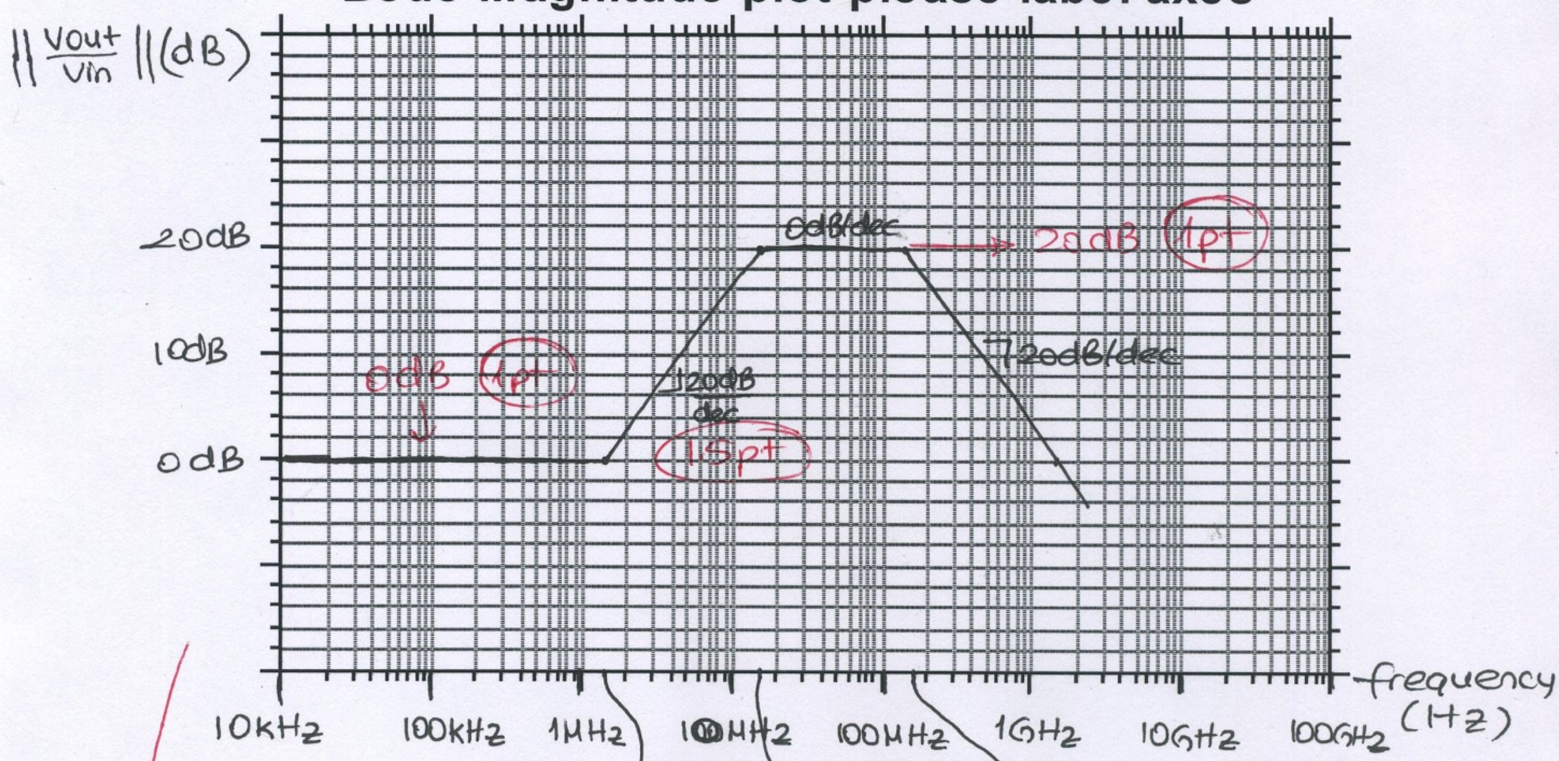
$f_{p2} = 160$ MHz

} 0.5 pt each = 1.5 pts

Part a, 10 points

Make an **accurate** asymptotic Bode plot of $\| H \|$, labelling and dimensioning axes and clearly labelling all slopes and all critical frequencies.

Bode Magnitude plot-please label axes



$f_z = 1.6$ MHz (1pt) $f_{p1} = 16$ MHz (1pt) $f_{p2} = 160$ MHz (1pt)

Correct scales & labels (2pts)

b exam

Part b, 10 points

Make an **accurate** root locus plot of H(s), labelling and dimensioning axes and clearly labelling all critical frequencies.

be sure to label axes with #s and units