ECE 2C Problem Set #1 Solutions

1.) LC filter passes 1 GHz component only → find compo[nent] with that

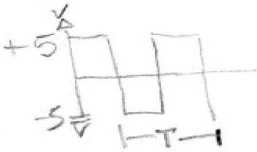
a.) Find ac voltage @ load

- definition of Fourier series

$$f(t) = a_0 + a_1\sqrt{2}\cos\left(1\cdot\frac{2\pi}{T}\cdot t\right) + a_2\sqrt{2}\cos\left(2\cdot\frac{2\pi}{T}\cdot t\right) + \dots + b_1\sqrt{2}\sin\left(1\cdot\frac{2\pi}{T}\cdot t\right) + b_2\sqrt{2}\sin\left(2\cdot\frac{2\pi}{T}\cdot t\right) + \dots$$

- we want to find coefficients $a_0, a_1, \dots, b_1, \dots$

$$a_0 = \frac{1}{T}\int_0^T V_{in}(t)\,dt$$

$V_{in} =$ 

$$T = \frac{1}{f} = \frac{1}{1\,\text{GHz}} = 1\times10^{-9}\,\text{s} = 1\,\text{ns}$$

$$a_0 = \frac{1}{1\times10^{-9}}\underbrace{\left[\int_0^{.5ns} 5\,dt + \int_{.5ns}^{1ns} -5\,dt\right]}_{0} = 0$$

$$a_1 = \frac{1}{T}\int_0^T V_{in}(t)\sqrt{2}\cos\left(n\cdot\frac{2\pi}{T}\, t\right)dt$$

$$= \frac{1}{1\times10^{-9}}\left[\int_0^{.5ns} 5\sqrt{2}\cos\left(1\cdot\frac{2\pi}{1ns}\cdot t\right)dt + \int_{.5ns}^{1ns} -5\sqrt{2}\cos\left(1 \dots\right.\right.$$

$$= \frac{1}{1\times10^{-9}}\left[\underbrace{\left[5\sqrt{2}\cdot\frac{1ns}{2\pi}\sin\left(1\cdot\frac{2\pi}{1ns}t\right)\right]_0^{.5ns}}_{0} - \underbrace{\left[5\sqrt{2}\cdot\frac{1ns}{2\pi}\sin\left(1 \dots\right.\right.}_{0}\right.$$

$$a_1 = 0$$

$$b_1 = \frac{1}{T}\int_0^T V_{in}(t)\sqrt{2}\sin\left(n\cdot\frac{2\pi}{T}\cdot t\right)dt$$

$$= \frac{1}{1ns}\left[\int_0^{.5ns} 5\sqrt{2}\sin\left(1\cdot\frac{2\pi}{1ns}\cdot t\right)dt + \int_{.5ns}^{1ns} -5\sqrt{2}\sin\left(1\cdot\frac{2\pi}{1ns}\cdot t\right)\right.$$

$$= \frac{1}{1ns}\left\{\left[-5\sqrt{2}\,\frac{1ns}{2\pi}\cos\left(1\cdot\frac{2\pi}{1ns}\cdot t\right)\right]_0^{.5ns} + \left[5\sqrt{2}\cdot\frac{1ns}{2\pi}\cos\left(1\cdot\frac{2\pi}{1ns}\cdot t\right)\right.\right.$$

$$= \frac{5\sqrt{2}}{2\pi}\left\{\underbrace{-\cos\left(1\cdot\frac{2\pi}{1ns}\cdot .5ns\right)}_{1} + \underbrace{\cos\left(\frac{2\pi}{1ns}\cdot 0\right)}_{1} + \underbrace{\cos\left(\frac{2\pi}{1ns}\cdot 1ns\right)}_{1} - \cos \dots\right.$$

$$b_1 = \frac{10\sqrt{2}}{\pi}$$

1.) a.) continued

$$V_{Load}(t) = \frac{10\sqrt{2}}{\pi} \cdot \sqrt{2} \sin\left(1 \cdot \frac{2\pi}{1ns} \cdot t\right)$$

$$\boxed{V_{Load}(t) = \frac{20}{\pi} V \sin\left(\frac{2\pi}{1ns} \cdot t\right)}$$

b.) $R_L = 50\,\Omega$

$P = ?$ $\quad P = \frac{(V_{rms})^2}{R}$

$$V_{rms} = \frac{20}{\pi\sqrt{2}} \Rightarrow P = \frac{(20)^2}{\pi^2 2R} = \frac{400}{\pi^2 100} = \boxed{\frac{4}{\pi^2} W}$$

c.) filter passes 3GHz signals

$$3\,GHz \Rightarrow n = 3$$

$b_3 = ?$

$$b_3 = \frac{1}{T}\int_0^T V_{in}(t)\sqrt{2} \sin\left(n \cdot \frac{2\pi}{T} \cdot t\right) dt$$

$$= \frac{1}{1ns}\left[\int_0^{.5ns} 5\sqrt{2} \sin\left(3 \cdot \frac{2\pi}{1ns} t\right) dt + \int_{.5ns}^{1ns} -5\sqrt{2} \sin\left(3 \cdot \frac{2\pi}{1ns} \cdot t\right) dt\right]$$

$$= \frac{1}{1ns}\left\{\left[-5\sqrt{2}\,\frac{1ns}{6\pi} \cos\left(\frac{6\pi}{1ns} \cdot t\right)\right]_0^{.5ns} + \left[5\sqrt{2}\,\frac{1ns}{6\pi} \cos\left(\frac{6\pi}{1ns} \cdot t\right)\right]_{.5ns}^{1ns}\right.$$

$$= \frac{5\sqrt{2}}{6\pi}\left[\underbrace{-\cos\left(\frac{6\pi}{1ns} \cdot .5ns\right)}_{1} + \underbrace{\cos\left(\frac{6\pi}{1ns} \cdot 0\right)}_{1} + \underbrace{\cos(6\pi)}_{1} - \underbrace{\cos\left(\frac{6\pi}{1ns} \cdot .5ns\right)}_{1}\right]$$

$$b_3 = \frac{10\sqrt{2}}{3\pi} \Rightarrow V_{Load}(t) = \frac{10\sqrt{2}}{3\pi} \cdot \sqrt{2} \sin\left(3 \cdot \frac{2\pi}{1ns} \cdot t\right)$$

$$\boxed{V_{Load}(t) = \frac{20}{3\pi} V \sin\left(\frac{6\pi}{1ns} \cdot t\right)}$$

$$\boxed{P = \frac{400}{9\pi^2 \cdot 2 \cdot 50} = \frac{4}{9\pi^2} W}$$

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########################################################################################
#Problem 2
#       (a) Plot the following functions over one cycle , where f =100 MHz.
#       Plot at least 32 points/waveform. To keep it from getting tedious,
#       please use Excel, Matlab, or some other computer program to calculate and
#       to plot.
########################################################################################

#Define Variables
        f=100000000  # Frequency = 100MHz
        To=1/f        #Period
        n=100         #number of points per waveform (problem says should be >32
        t=seq(from=0, to= To, by= To/n) #discrete points in time for one cycle

#Enter functions (given) and plot them
        V1 = sin(2*pi*f*t)
        plot(x=t, y=V1)        #Figure 1a attached

        V3 = V1 + (1/3)*sin(3*2*pi*f*t)
        plot(x=t, y=V3)        #Figure 1b

        V5 = V3 + (1/5)*sin(5*2*pi*f*t)
        plot(x=t, y=V5)        #Figure 1c

        V7 = V5 + (1/7)*sin(7*2*pi*f*t)
        plot(x=t, y=V7)        #Figure 1d

        V9 = V7 + (1/9)*sin(9*2*pi*f*t)
        plot(x=t, y=V9)        #Figure 1e
```

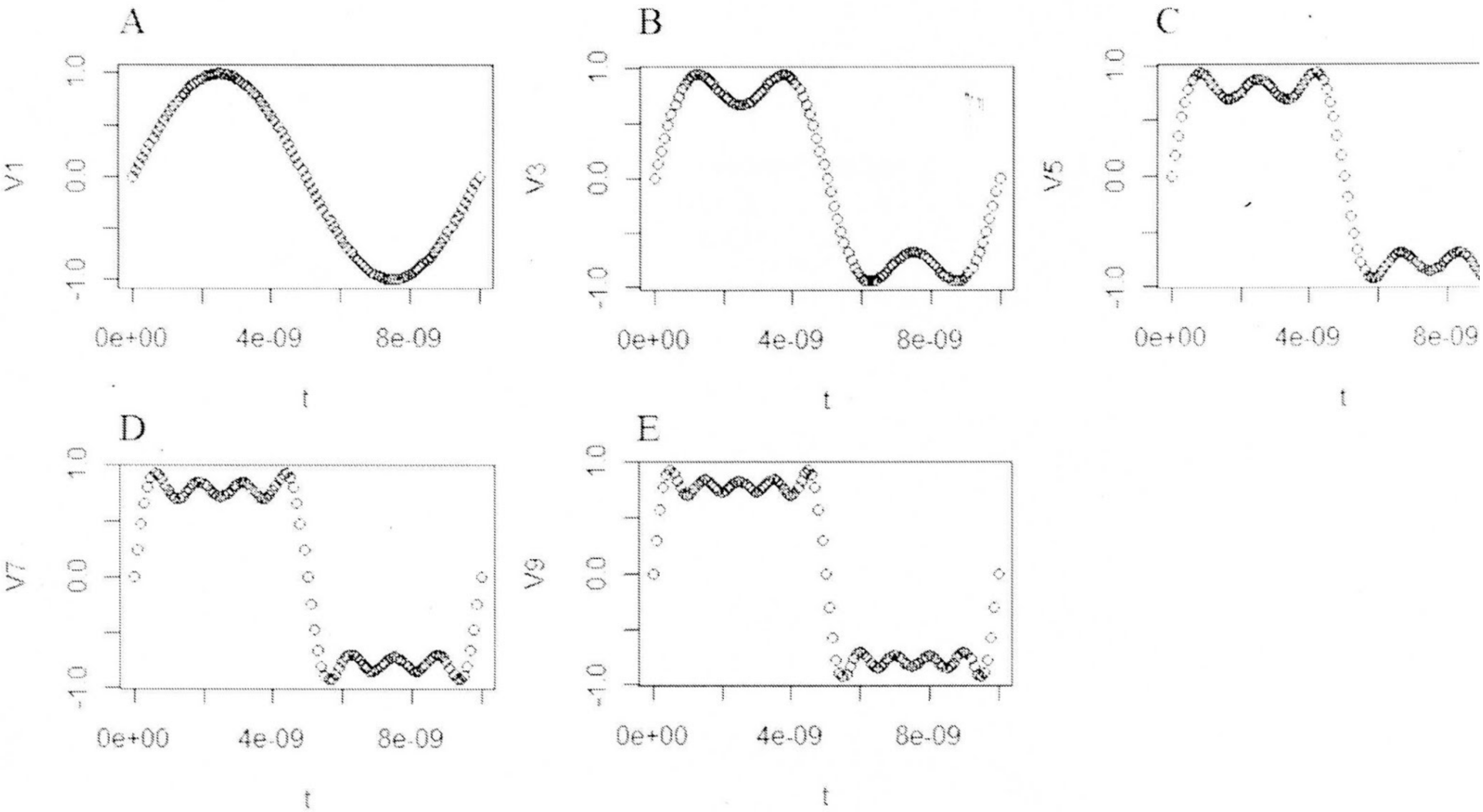

Figure 1: Plots for problem 2

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##########################################################################################
#       (b)what do you think Vn(t) becomes when n is very large ?
#
#       As n gets very large Vn(t) will become a square wave.
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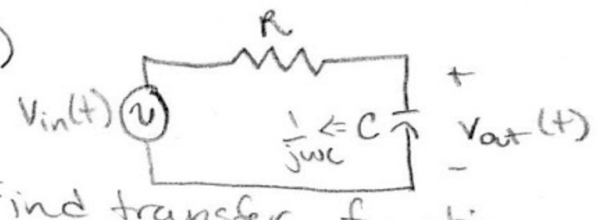
ECE 2C Problem Set #1 Solutions P

a.)



$R = 100\Omega$
$RC = 1ns$
$f = 100MHz$
$\omega_0 = 2\pi f = 2\pi \cdot 100MHz$

• Find transfer function
Voltage Divider:

$$V_{out} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} \cdot V_{in} = \frac{1}{j\omega C\left(\frac{1}{j\omega C} + R\right)} \cdot V_{in} = \frac{1}{1 + j\omega RC} \cdot V_{in}$$

$$T(j\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC}$$

• $$|T(j\omega)| = \left(\frac{1}{1 + j\omega RC}\right) \cdot \left(\frac{1}{1 - j\omega RC}\right) = \sqrt{\frac{1}{1 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (n\omega_0 RC)}}$$

• $$\phi_T = -\tan^{-1}\left(\frac{\omega RC}{1}\right) = -\tan^{-1}\left(\frac{n\omega_0 RC}{1}\right)$$

Find Amplitude and Phase of V_{in}

$V_{in}(t) = 1V \cdot \sin(2\pi f t)$

$|V_{in}| = 1V$

$\phi_{V_{in}} = -\pi/2$

• Find Amplitude and phase of V_{out}

$$|V_{out}| = |V_{in}| \cdot |T(j\omega)| = 1V \cdot \frac{1}{\sqrt{1 + (1 \cdot 2\pi \cdot 100MHz \cdot 1ns)^2}} =$$

$$\phi_{V_{out}} = \phi_{V_{in}} + \phi_T = -\pi/2 - \tan^{-1}(n\omega_0 RC) = -2.13 \text{ rad}$$

$$V_{out}(t) = 0.85V \cdot \cos(n\omega_0 t + \phi_{V_{out}})$$
$$= 0.85V \cdot \cos(1 \cdot 2\pi \cdot 100MHz\, t - 2.13 \text{ rad})$$

3.) continued

b.) $V_{in}(t) = \frac{1}{3} V \cdot \sin(3 \cdot 2\pi f t)$

$|V_{in}| = \frac{1}{3} \quad \phi_{V_{in}} = -\pi/2 \quad n = 3$

$|V_{out}| = \frac{1 \cdot V}{3} \cdot \frac{1}{1 + (3 \cdot 2\pi \cdot 100\,MHz \cdot 1ns)^2} = 0.156V$

$\phi_{out} = -\pi/2 - \tan^{-1}(3 \cdot 2\pi \cdot 100MHz \cdot 1ns) = -2.65 rad$

$V_{out} = 0.156V \cdot \cos(3 \cdot 2\pi \cdot 100MHz \cdot t - 2.65 rad)$

c.) $V_{in}(t) = \frac{1}{5} \cdot V \cdot \sin(5 \cdot 2\pi f t)$

$|V_{in}| = \frac{1 \cdot V}{5} \quad \phi_{V_{in}} = -\pi/2 \quad n = 5$

$|V_{out}| = \frac{1 \cdot V}{5} \cdot \frac{1}{1 + (5 \cdot 2\pi \cdot 100MHz \cdot 1ns)^2} = 0.061V$

$\phi_{out} = -\pi/2 - \tan^{-1}(5 \cdot 2\pi \cdot 100MHz \cdot 1ns) = -2.83 rad$

$V_{out} = 0.061V \cdot \cos(5 \cdot 2\pi \cdot 100MHz \cdot t - 2.83 rad)$

d.) $V_{in}(t) = \frac{1V}{7} \sin(7 \cdot 2\pi f t)$

$|V_{in}| = \frac{1}{7} V \quad \phi_{V_{in}} = -\pi/2 \quad n = 7$

$|V_{out}| = \frac{1 \cdot V}{7} \cdot \frac{1}{1 + (7 \cdot 2\pi \cdot 100MHz \cdot 1ns)^2} = 0.032V$

$\phi_{V_{out}} = -\frac{\pi}{2} - \tan^{-1}(7 \cdot 2\pi \cdot 100MHz \cdot 1ns) = -2.91 rad$

$V_{out} = 0.032V \cdot \cos(7 \cdot 2\pi \cdot 100MHz \cdot t - 2.91 rad)$

e.) $V_{in}(t) = \frac{1V}{9} \sin(9 \cdot 2\pi f t)$

$|V_{in}| = \frac{1}{9} V \quad \phi_{in} = -\pi/2 \quad n = 9$

$|V_{out}| = \frac{1V}{9} \cdot \frac{1}{1 + (9 \cdot 2\pi \cdot 100\,MHz \cdot 1ns)^2} = 0.019V$

$\phi_{V_{out}} = -\frac{\pi}{2} - \tan^{-1}(9 \cdot 2\pi \cdot 100MHz \cdot 1ns) = -2.97 rad$

$V_{out} = 0.019V \cdot \cos(9 \cdot 2\pi \cdot 100MHz \cdot t - 2.97 rad)$

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##########################################################################################
#       Problem 3: R=100 Ohms and RC= 1nS. f=100 MHz.(a) Using Phasor analysis,
#       compute note the correction ->Vout(t) if Vin (t) =1V ×sin(2pft) . <---
#       (b) repeat with Vin (t) =(1/ 3)×1V ×sin(3*2pft)
#       (c) repeat with Vin (t) =(1/ 5)×1V ×sin(5*2pft)
#       (d) repeat with Vin (t) =(1/ 7)×1V ×sin(7*2pft)
#       (e) repeat with Vin (t) =(1/ 9)×1V ×sin(9*2pft)
#       use Excel, Matlab, or some other computer program to calculate and to plot
#       the sum of the answers you found in parts (a-e).
##########################################################################################

        #Define values
                f=100000000  # Frequency = 100MHz
                R = 100
                RC = 1e-9
                Wo=2*pi*f
                m=seq(from=1, to =9, by=2)
                To=1/f #Period
                n=100           #number of points per waveform
                t=seq(from=0, to= To, by= To/n)      #discrete points in time for one cycle


        #Define values of amplitude and phase for Vin
                AmpVin= 1/m
                PhaseVin=rep(-pi/2, 5)

        #Calculate amplitude and phase for Transfer function
                AmpT = 1/sqrt((1+(m*Wo*RC)^2))
                PhaseT = -atan(m*Wo*RC)

        #Calculate amplitude and phase for Vout
                AmpVout= rep(0, 5)
                PhaseVout= rep(0,5)
                for (i in c(1:5)){
                        AmpVout[i] = AmpT[i]*AmpVin[i]
                        PhaseVout[i] = PhaseT[i] + PhaseVin[i]
                }#end for

        #Plug in the appropriate amplitude and phase
                Vout1= AmpVout[1]*cos(1*Wo*t+ PhaseVout[1])
                Vout2= AmpVout[2]*cos(3*Wo*t+ PhaseVout[2])
                Vout3= AmpVout[3]*cos(5*Wo*t+ PhaseVout[3])
                Vout4= AmpVout[4]*cos(7*Wo*t+ PhaseVout[4])
                Vout5= AmpVout[5]*cos(9*Wo*t+ PhaseVout[5])

        #Add up all the parts and plot
                VoutTotal= Vout1 + Vout2 +Vout3 + Vout4 + Vout5
                plot (y=VoutTotal, x=t) # See figure 2
```

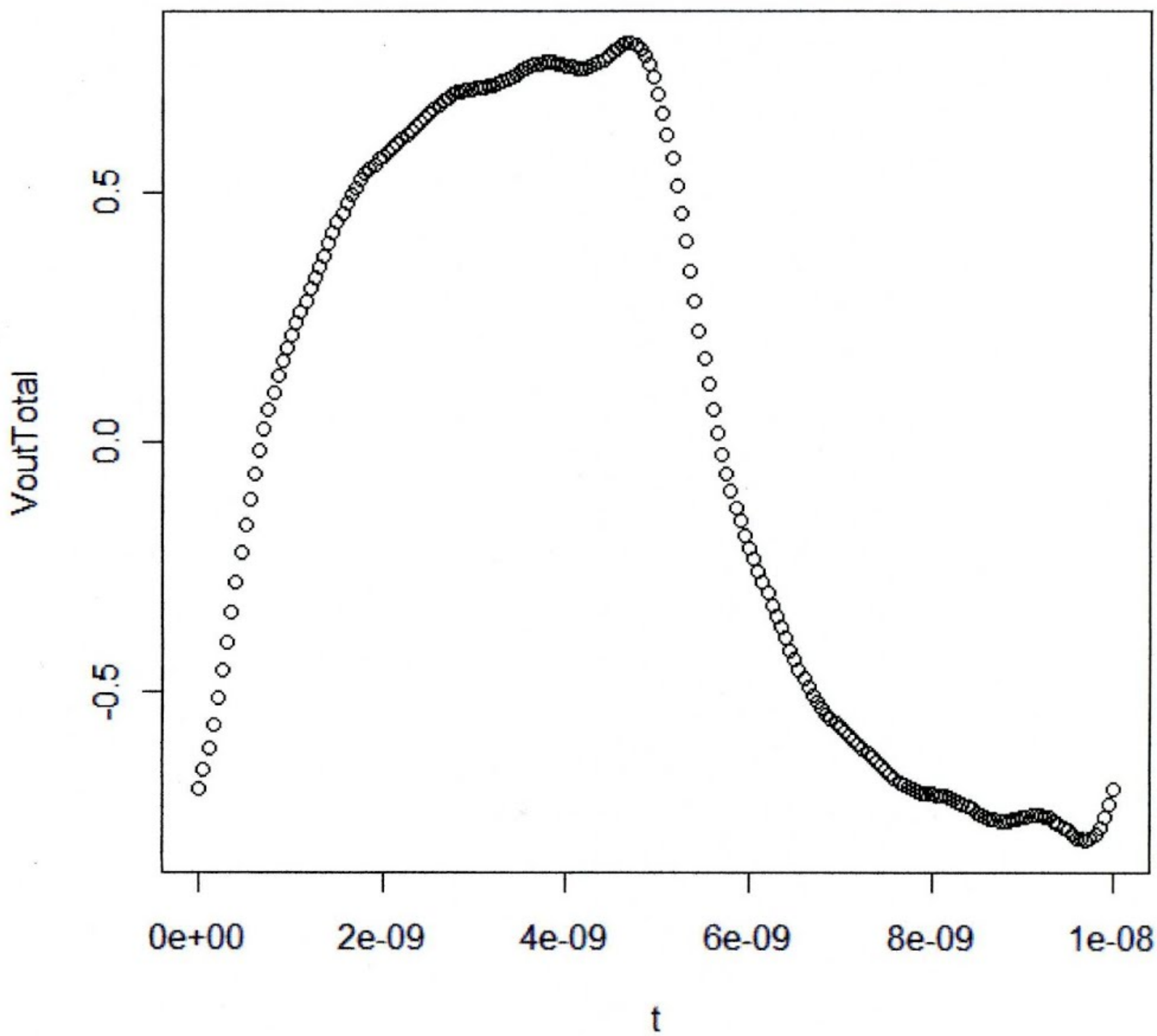

Figure 2: Plot for problem 3

Note: (a) This is an illustration of working a circuit using a square wave input, by breaking that square wave down into an input set of sine waves (Fourier series as we did in Problem 2), finding the propagation of each to the output, and then adding them up. In other words, using Fourier methods.
(b) The solution is approaching the transient solution, which is a set of charging and discharging exponentials, each with time constant tau=RC.

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############################################################################################
#       Problem 4: (a) Plot the following functions over one cycle , where f =100
#       MHz. Plot at least 32 points/waveform. To keep it from getting tedious,
#       please use Excel, Matlab, or some other computer program to calculate and
#       to plot.
############################################################################################

    #Define Variables (same as Problem 2)
        f=100000000  # Frequency = 100MHz
        To=1/f       #Period
        n=100        #number of points per waveform (problem says should be >32
        t=seq(from=0, to= To, by= To/n) #discrete points in time for one cycle

    #Enter functions (given) and plot them I will call V1 VA to distinguish it
    #from problem 2
        VA = sin(2*pi*f*t)
        plot(x=t, y=VA)        #Figure 3a attached

        VB = VA - (1/2)*sin(2*2*pi*f*t)
        plot(x=t, y=VB)        #Figure 3b

        VC = VB + (1/3)*sin(3*2*pi*f*t)
        plot(x=t, y=VC)        #Figure 3c

        VD = VC - (1/4)*sin(4*2*pi*f*t)
        plot(x=t, y=VD)        #Figure 3d
```

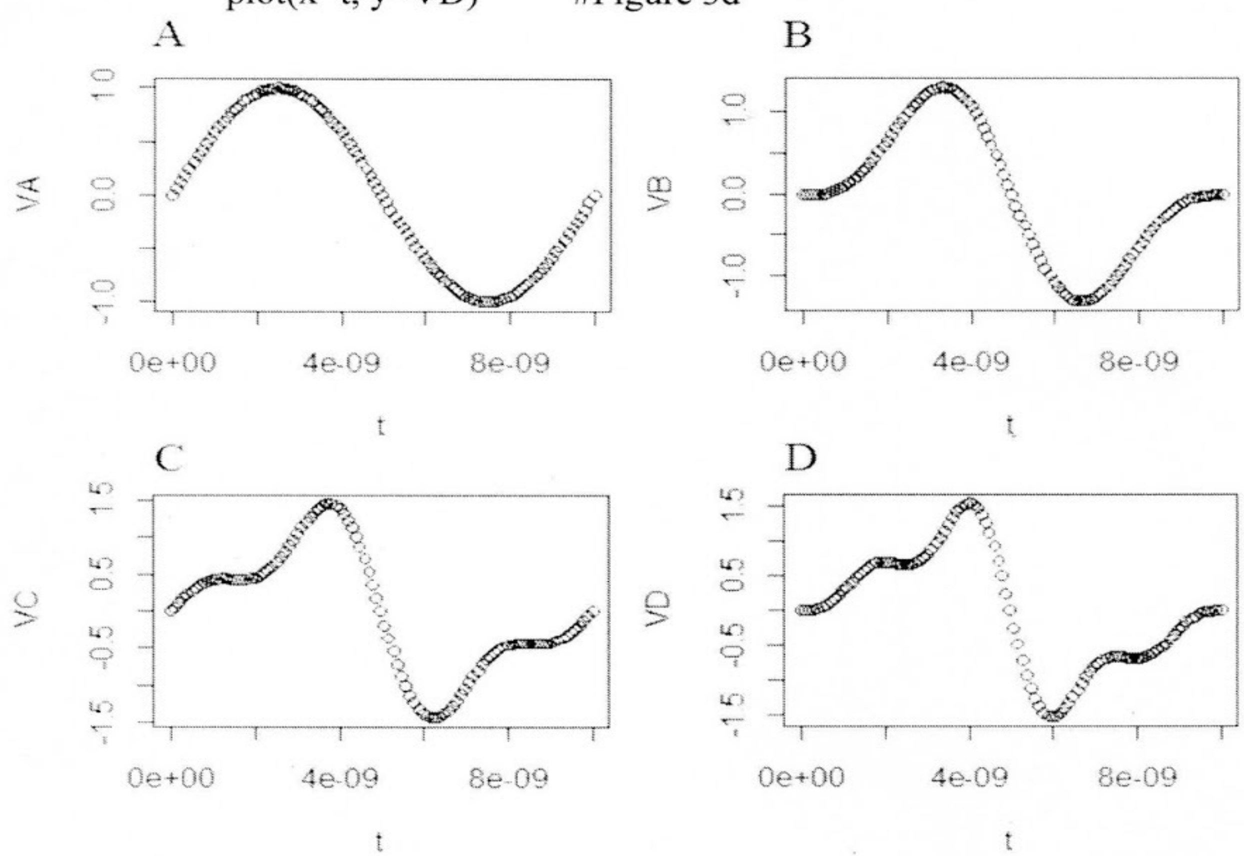

Figure 3: Plots for problem 4

##
#	(b)what do you think Vn(t) becomes when n is very large ?
#
#	As n increases this function is approaching a sawtooth wave.
##

5.) a.) $V_{sig}(t) = a_1 V_1(t) + a_2 V_2(t) + a_3 V_3(t) + a_4 V_4(t)$

$$a_1 = V_{sig} \cdot V_1 = \frac{1}{T}\int_0^T V_{sig} \cdot V_1 \, dt$$

$$= \frac{1}{4ns}\left[\int_0^{1ns} 1 \cdot 1 \, dt + \int_{1ns}^{2ns} -1 \cdot 1 \, dt + \int_{2ns}^{3ns} 1 \cdot 1 \, dt + \int_{3ns}^{4ns} 0 \cdot 1 \, dt\right]$$

$$= \frac{1}{4ns}\left\{ [t]_0^{1ns} + [-t]_{1ns}^{2ns} + [t]_{2ns}^{3ns} \right\}$$

$$= \frac{1}{4ns}(1ns + -2ns + 1ns + 3ns - 2ns) = \boxed{\frac{1}{4} = a_1}$$

$$a_2 = V_{sig} \cdot V_2 = \frac{1}{4}\left[\int_0^1 1 \cdot 1 \, dt + \int_1^2 -1 \cdot 1 \, dt + \int_2^3 1 \cdot -1 \, dt + \int_3^4 0 \cdot -1 \, dt\right]$$

$$= \frac{1}{4}\left\{ [t]_0^1 + [-t]_1^2 + [-t]_2^3 \right\}$$

$$= \frac{1}{4}\left[1 \; -2 + 1 \; -3 + 2\right] = \boxed{-\frac{1}{4} = a_2}$$

$$a_3 = V_{sig} \cdot V_3 = \frac{1}{4}\left[\int_0^1 1 \cdot 1 \, dt + \int_1^2 -1 \cdot -1 \, dt + \int_2^3 1 \cdot -1 \, dt + \int_3^4 0 \cdot 1 \, dt\right.$$

$$= \frac{1}{4}\left\{ [t]_0^1 + [t]_1^2 + [-t]_2^3 \right\}$$

$$= \frac{1}{4}\left[1 + 2 - 1 \; -3 + 2\right] = \boxed{\frac{1}{4} = a_3}$$

$$a_4 = V_{sig} \cdot V_4 = \frac{1}{4}\left[\int_0^1 1 \cdot 1 \, dt + \int_1^2 -1 \cdot -1 \, dt + \int_2^3 1 \cdot 1 \, dt + \int_3^4 0 \cdot -1 \, dt\right]$$

$$= \frac{1}{4}\left\{ [t]_0^1 + [t]_1^2 + [t]_2^3 \right\} = \frac{1}{4}\left[1 + 2 - 1 + 3 - 2\right] = \boxed{\frac{3}{4} = a_4}$$

b.) $$V_3 \cdot V_4 = \frac{1}{4}\left[\int_0^1 1 \cdot 1 \, dt + \int_1^2 -1 \cdot -1 \, dt + \int_2^3 -1 \cdot 1 \, dt + \int_3^4 1 \cdot -1 \, dt\right]$$

$$= \frac{1}{4}\left\{ [t]_0^1 + [t]_1^2 + [-t]_2^3 + [-t]_3^4 \right\}$$

$$= \frac{1}{4}\left[1 + 2 - 1 \; -3 + 2 \; -4 + 3\right] = 0 \checkmark$$

V_3 and V_4 are orthogonal so their dot product should be