Question Paper A

1) a

$V_{RF}(t) = \cos(\omega_{LO}t) \times 200mV \sin(2\pi 100 t) - 200mV \sin(\omega_{LO}t)\cos(2\pi 200 t)$

$\cos A \sin B = \frac{\sin(A+B)}{2} - \frac{\sin(A-B)}{2}$

$\sin A \cos B = \frac{\sin(A+B)}{2} + \frac{\sin(A-B)}{2}$

$V_{RF}(t) = 100mV \Big[\sin(\omega_{LO} + \omega_I)t - \sin(\omega_{LO} - \omega_I)t - \sin(\omega_{LO} + \omega_Q)t$

$- \sin(\omega_{LO} - \omega_Q)t \Big]$

$\omega_{LO} = 2\pi \times 10^9$

$\omega_I = 2\pi \times 100$

$\omega_Q = 2\pi \times 200$

Frequency	10^9+100	10^9-100	10^9+200	10^9-200
Amp	100 mV	100 mV	100 mV	100 mV
Phase	-90	+90	+90	+90

1(b) $V_{RF}(t) = V_I(t)\cos(\omega_{LO}t) - V_Q(t)\sin(\omega_{LO}t)$

$V_d(t) = V_I(t)\cos^2(\omega_{LO}t) - V_Q(t)\sin(\omega_{LO}t)\cos(\omega_{LO}t)$

$\cos^2\theta = \frac{1}{2}\left[1 + \cos 2\theta\right]$

$\cos\theta \sin\theta = \frac{1}{2}\sin 2\theta$

$$V_d(t) = V_I(t)\left(\frac{1+\cos 2\omega_{LO}t}{2}\right) - \frac{V_Q(t)\sin 2\omega_{LO}t}{2}$$

Neglect higher frequency terms

$$V_d(t) = \frac{V_I(t)}{2} = \frac{1}{2} \times 200mV \quad \sin 2\pi \times 100t$$

$$= 100mV \quad \sin(2\pi 100t)$$

c)

$$V_{LO}(t) = \sqrt{2}\cos(\omega_{LO}t + \pi/4)$$

$$= \cos\omega_{LO}t - \sin\omega_{LO}t$$

$$V_{RF}(t) = V_I(t)\cos(\omega_{LO}t) - V_Q(t)\sin(\omega_{LO}t)$$

$$V_D(t) = V_I(t)\left[\cos^2(\omega_{LO}t) - \cos(\omega_{LO}t)\sin(\omega_{LO}t)\right] +$$

$$V_Q(t)\left[\sin^2(\omega_{LO}t) - \sin(\omega_{LO}\right.$$

$$\cos(\omega_{LO}t$$

$$\sin^2\theta = \frac{1}{2}\left[1-\cos 2\theta\right]$$

Neglect the higher frequency terms

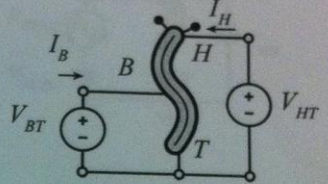
$$V_d(t) = \frac{V_I(t) + V_Q(t)}{2}$$

Problem 2, 30 points

Neurology researchers at UC Santa Cruz have discovered that applying small voltages between the belly (B) and tail (T) of a banana slug results in modulation of the currents between head (H) and tail (T). They find

$I_B = K_1 V_{BT}^2$ where $K_1 = 1\mu A/V^2$ and

$I_H = K_2 V_{HT} / V_{BT}$ where $K_2 = 1\mu A$



part a, 10 points

Assuming DC voltages $V_{BT,DC}$=1 V and $V_{HT,DC}$=2V, draw below a small-signal equivalent circuit of the slug. Give all small-signal parameters.

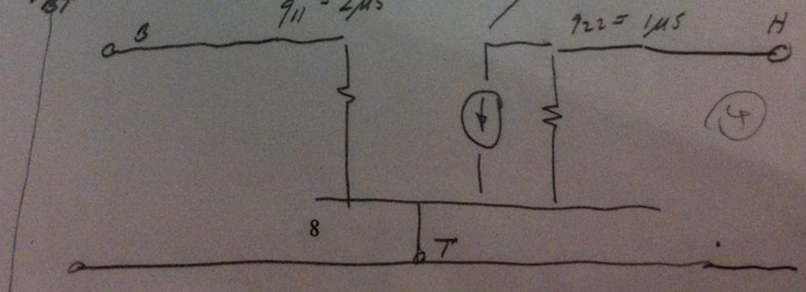
$I_B = K_1 V_{BT}^2$ $\qquad$ $I_H = K_2 V_{HT}/V_{BT}$

$\frac{\partial I_B}{\partial V_{BT}} = 2K_1 V_{BT} = 2 \cdot 1\mu A/V^2 \cdot 1V = 2\mu A/V = 2\mu S = g_{11}$ $\quad$ 2

$G_{21} : \frac{\partial I_H}{\partial V_{BT}} = -\frac{K_2 V_{HT}}{V_{BT}^2} = \frac{-1\mu A\,(2V)}{(1V)^2} = -2\mu A/V = -2\mu S = g_{21}$ $\quad$ 2

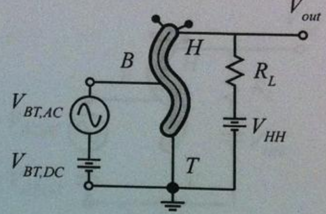
$G_{22} = \frac{\partial I_H}{\partial V_{HT}} = \frac{K_2}{V_{BT}} = \frac{1\mu A}{1V} = 1\mu S = G_{22}$ $\quad$ 2

$g_{21} \cdot V_{BT}$, where $g_{21} = -2\mu S$



part b, 10 points

We now set $V_{BT,DC}=1$ V and $R_L=1$ MΩ. We want the DC value of V_{out} to be 2.0 Volts. Find the necessary value of V_{HH}. Find the DC currents entering the H and B electrodes.



V_{HH}= 4V.

DC current into H electrode = 2μA

DC current into B electrode = 1μA

$$I_H = K_2 V_{HT}/V_{BT} = 1\mu A \cdot 2V/1V = 2\mu A.$$

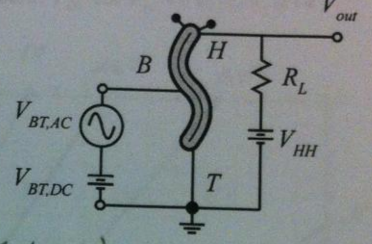
Voltage drop across R_L = $2\mu A \cdot 1M\Omega = 2V$

$$V_{HH} = V_H + 2V = 2V + 2V = 4V$$

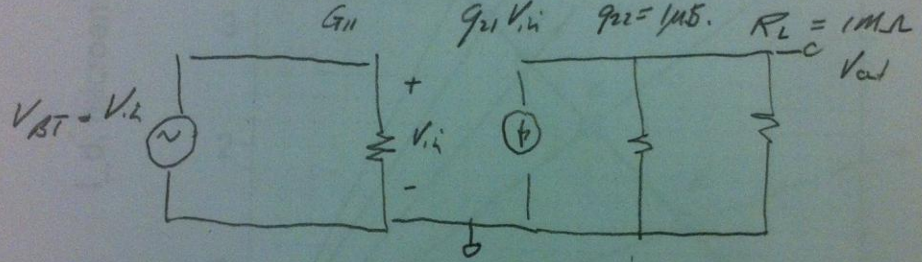
$$I_B = K_1 V_{BT}^2 = 1\mu A/V^2 \cdot 1V^2 = 1\mu A$$

part c, 10 points

Continuing with the values you have found in parts (a) and (b), a 1 kHz sine wave of 1 mV RMS amplitude is applied, as shown, to the belly. Find the resulting AC output voltage $V_{out}(t)$



$V_{out}(t) =$ $1\,mV \cdot \sqrt{2} \cdot \sin(2\pi \cdot 1kHz \cdot t)$



$g_{21} = -2\mu S$

$R_{22} = 1/g_{22} = 1M\Omega$

$1M\Omega \parallel 1M\Omega = 500k\Omega$

$V_{out} = -g_{21} V_{in} \cdot (500k\Omega)$

$= 2\mu S \cdot 500k\Omega \cdot V_{in}$

$= 1 V_{in}$..

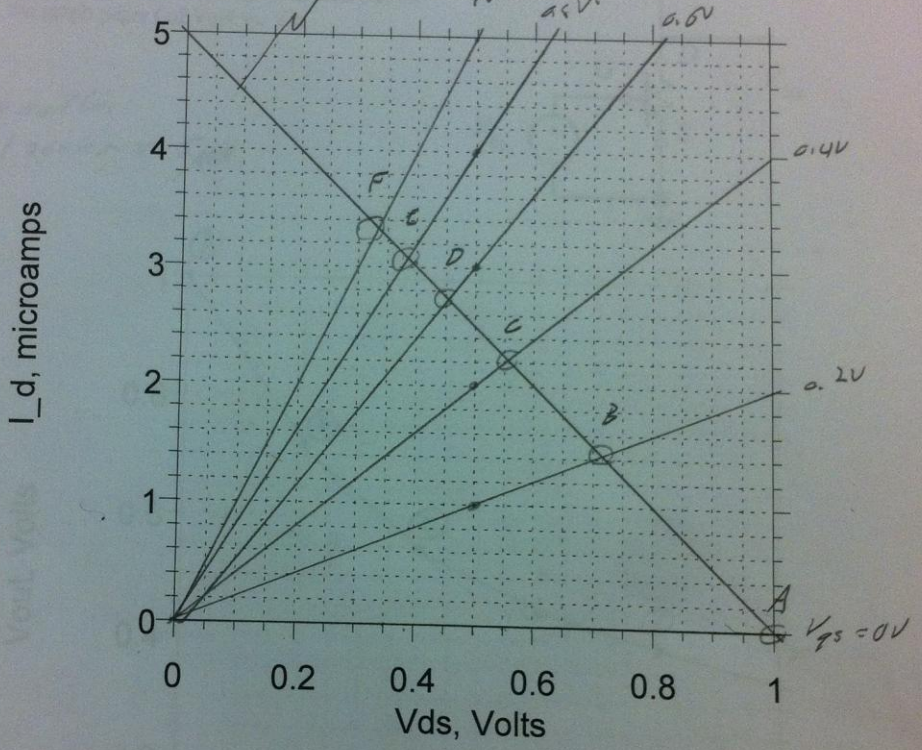
prob 3.

$I_D = \frac{10\mu A}{V^2} \cdot V_{gs} V_{DS}$

Part a, 5 points

loadline.

On the graph paper below. First plot I_D as a function of V_{DS} for V_{gs} = 0V, 0.2V, 0.4V, 0.6V, 0.8V, and 1.0V.

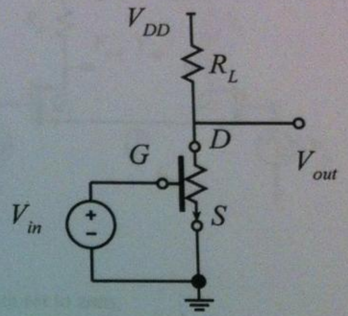


I_D = 0A for V_{gs} = 0V

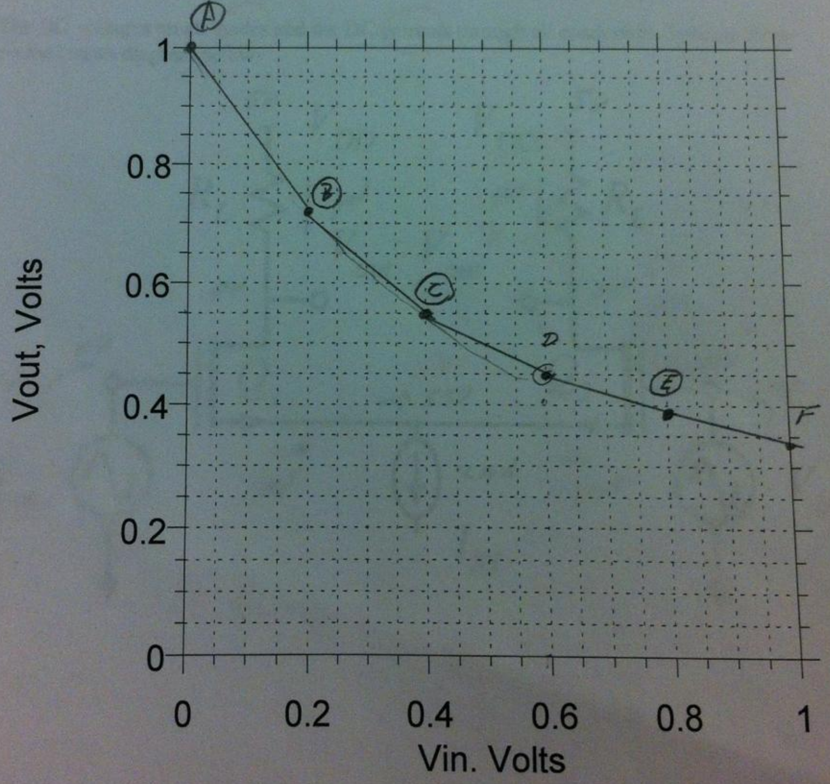
= 2μA/V " "	= 0.2	1	@ V_{DS} = 1/2 V
= 4 μA	0.4	2	"
= 6 μA	0.6	3	"
8 μA	0.8	4	"
10 μA/V.	1.0	5 μA	"

Part b, 10 points

We now connect the device, as shown to a V_{DD}=1.0 V power supply and R_L= 200 kΩ load resistor. Add the loadline associated with RL to the previous page. Use this to make a plot on the graph paper (of Vout vs. Vin.



Loadline:
1V/200kΩ = 5μA.



Problem 4, 25 points

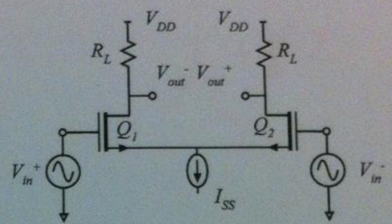
Q1 and Q2 are velocity-limited NFETs, i.e.

$I_d = C_{ox} v_{sat} W_g (V_{gs} - V_{th})(1 + \lambda V_{ds})$ for $V_D > V_g - V_{th}$.

We have

$\lambda = 0\text{V}^{-1}$, $C_{ox} v_{sat} W_g = 4\text{mA/V}$,

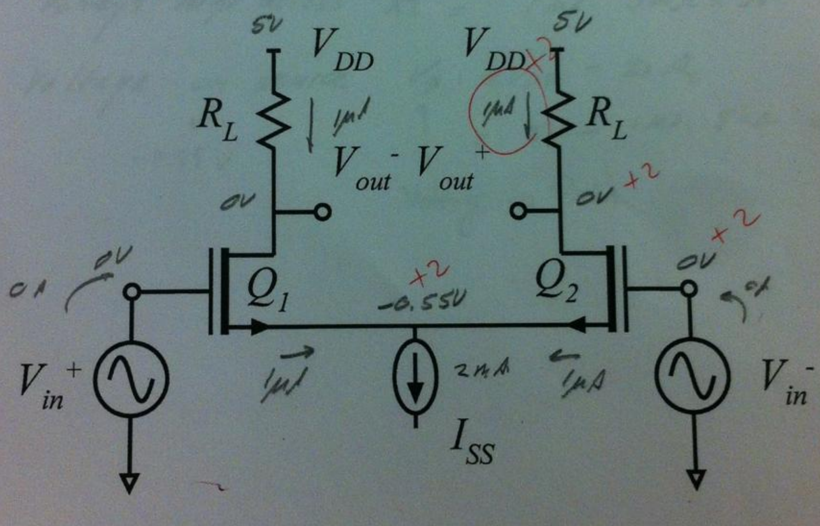
$V_{th} = 0.3\text{V}$, $V_{DD} = 5\text{V}$, $R_L = 5\text{k}\Omega$ and I_{ss}=2 mA.



part a, 10 points

Under DC bias conditions, the input voltages are both set to zero.

The DC voltages on all nodes and the DC currents through all conductors. Indicate these on the circuit diagram below.



$I_D = (4\,mA/V)(V_{gs} - 0.3V)$.

But $I_D = 1mA$

$+2$

$1mA = \frac{4\,mA}{V}(V_{gs} - V_{tl})$

$V_{gs} - V_{tl} = 0.25V$

$V_{gs} = 0.55V$. But since $V_g = 0V$

$V_s = -0.55V$.

Voltage drop across $R_L = 1mA \cdot 5k\Omega = 5V$

Voltage on source $V_D = V_{DD} - I_D R_L$

$-0.55V$

$= 5V - 1mA \cdot 5k\Omega = 0V$

Voltage on Drain

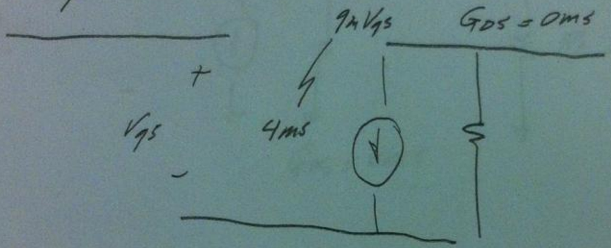
part b, 5 points

Draw the *FET* small signal equivalent circuit model, giving the values of all small-signal parameters.

$$I_D = (4mA/V)(V_{gs} - 0.5V)$$

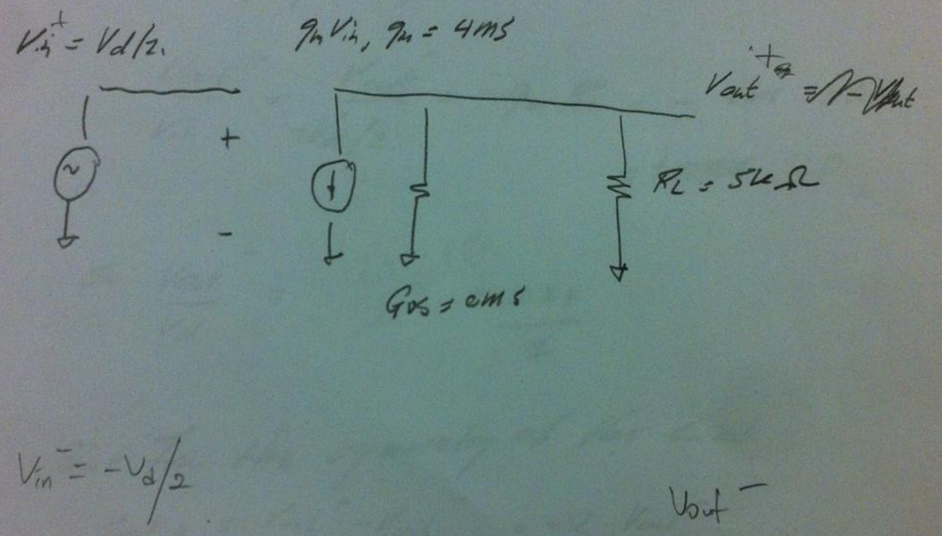
$$G_{DS} = g_{22} = \partial I_D / \partial V_{DS} = 0$$

$$g_m = g_{21} = \partial I_D / \partial V_{gs} = 4\,mA/V. = 4\,mS.$$



part c, 7 points

Now, setting $V_{in}^{+} = V_{in,d}/2$ and $V_{in}^{-} = -V_{in,d}/2$, draw a small-signal equivalent circuit of the *amplifier*. Please feel free to exploit the natural symmetry of the circuit to simplify the diagram.



part d, 8 points

$V_{in,d}$ is a 1 mV (peak) amplitude sine wave at 1 kHz. Find the AC small-signal differential output voltage $V_{out,D}(t) = V_{out}^{+} - V_{out}^{-}$.

from page 19,

$$\frac{V_{out}^-}{V_{in}} = \frac{V_{out}^-}{-V_d/2} = -g_m R_L = -4mS \cdot 5k\Omega$$

$$= + \cancel{1.25}\ 20$$

$$\text{so } \frac{V_{out}^-}{V_d} = \frac{20}{2} = 10 \quad + \frac{\cancel{1.25}}{2}$$

But from the symmetry of the circuit

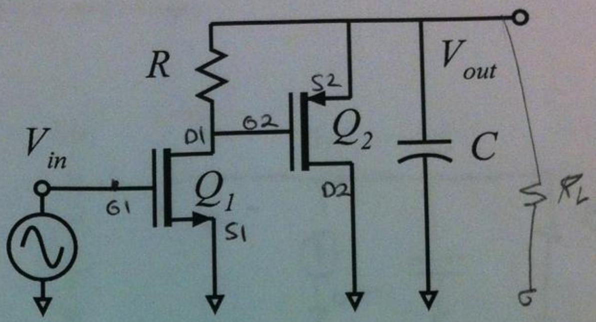
$$V_{out,D} = V_{out}^+ - V_{out}^- = -2 \cdot V_{out}^-$$

$$\boxed{\text{so } \frac{V_{out\,D}}{V_{in}} = \cancel{-2.25}\ \mp 20}$$

Problem 5, 25 points

You will be working with the circuit at right. Ignore DC bias. You don't need it.

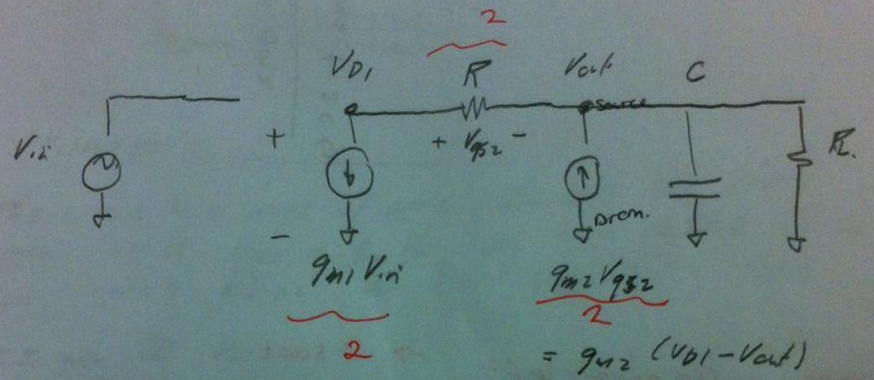
The transistors Q1 and Q2 have transconductances g_{m1} and g_{m2}, ***both nonzero,*** but $G_{ds1} = G_{ds2}$ **=0 mS.** The transistors have no gate-source or gate-drain capacitances.



$$\frac{R_L\left(g_{m1} + g_{m1} g_{m2} R\right)}{sCR_L + 1}$$

$$\frac{g_{m1} R_L\left(1 + g_{m2} R\right)}{1 + sCR_L}.$$

Part a, 8 points
replacing the transistors with their small-signal models, draw a small-signal equivalent circuit, labeling all elements and all control voltages.



2

Vgs2 → VD1 −1

Vgs1 & Vgs2 not shown −3

Part b, 12 points

Compute $V_{out}(s)/V_{in}(s)$: The answer must be in one of these two standard forms:

$$\frac{V_{out}(s)}{V_{in}(s)} = \left.\frac{V_{out}}{V_{in}}\right|_{\substack{\text{low-frequency}\\ \text{value}}} \times \frac{1 + b_1 s + b_2 s^2 + \dots}{1 + a_1 s + a_2 s^2 + \dots} \quad \text{or}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \left.\frac{V_{out}}{V_{in}}\right|_{\substack{\text{low-frequency}\\ \text{value}}} \times \frac{(1 - s/s_{z1})(1 - s/s_{z2})\dots}{(1 - s/s_{p1})(1 - s/s_{p2})\dots}$$

$V_{out}(s)/V_{in}(s) =$ ____________________________

There are two ways of working this, both get credit

one: write $V_{gs2} = g_{m1} V_{in}$ and write $\Sigma I = 0$ @ V_{out}

two: write $\Sigma I = 0$ @ V_{D1} and V_{out}.

$\Sigma I = 0$ @ V_{D1}

Eq 1 $\quad g_{m1} V_{in1} + (V_{D1} - V_{out})/R = 0$ +4

$\Sigma I = 0$ @ V_{out}:

Eq 2 $\quad \boxed{\frac{V_{out} - V_{D1}}{R} + (-1)\, g_{m2}(V_{D1} - V_{out}) + sC V_{out} + V_{out}/R_L = 0}$

+4

If $V_{D1} - V_{out}$ is wrong -2

Now use

$$\frac{V_{D1} - V_{out}}{R} = -g_{m1} V_{in}$$

or $V_{D1} - V_{out} = -g_{m1} R \cdot V_{in}$

so, substituting into Eq. 2;

+2

$$g_{m1} V_{in} - g_{m2} (-g_{m1} R) V_{in} + sC V_{out} + V_{out} G_L = 0$$

$$V_{in} [g_{m1} (1 + g_{m2} R_L)] + (G_L + sC) V_{out} = 0$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_{m1}(1 + g_{m2} R)}{G_L + sC} = \frac{-g_{m1} R_L (1 + g_{m2} R)}{1 + sCR_L}$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{-g_{m1} R_L (1 + g_{m2} R)}{1 + sCR_L}}$$

+2

✓

$$\frac{-g_{m1}(1+g_{m2}R)}{} \quad \frac{-g_{m1} \left(\frac{1}{R} + g_{m2} \right)}{(1 + sC/G_L) G}$$

Part c, 5 points

Now $g_{m1} = g_{m2} = 1$ ms, $R = 1$ kΩ, $C = 1$ pF. $R_L = 100\Omega$

Give the frequencies, in Hz, of any and all poles and zeros. Clearly indicate if any lie in the right half of the S-plane → None de

1st pole frequency= 1.59GHz, 2nd pole frequency= N/A,
3rd pole frequency= N/A

1st zero frequency= N/A, 2nd zero frequency= N/A,
3rd zero frequency= N/A

$$\frac{V_{out}}{V_{in}} = -g_{m1}(1+\underbrace{g_{m2}R}_{2})\cdot R_L \frac{1}{1+sCR_L}$$

1ms, 1ms, 1kΩ, 100Ω, 1pF, 100Ω

$$= -0.2 \cdot \frac{1}{1+s\cdot 100ps}$$

$\tau = 100ps$

$$f_{pole} = \frac{1}{2\pi\tau} = 1.59GHz \checkmark$$

If it has a small mistake in the trasfer fnc (-1)

$\boxed{-2}$ for minus

$\boxed{-3}$ rad/sec.

Problem 6, 20 points

We have a circuit for which

$$\frac{V_{out}(s)}{V_{in}(s)} = H(s) = H_{DC} \times \frac{1}{1 + s(2\zeta/\omega_n) + s^2/\omega_n^2}.$$

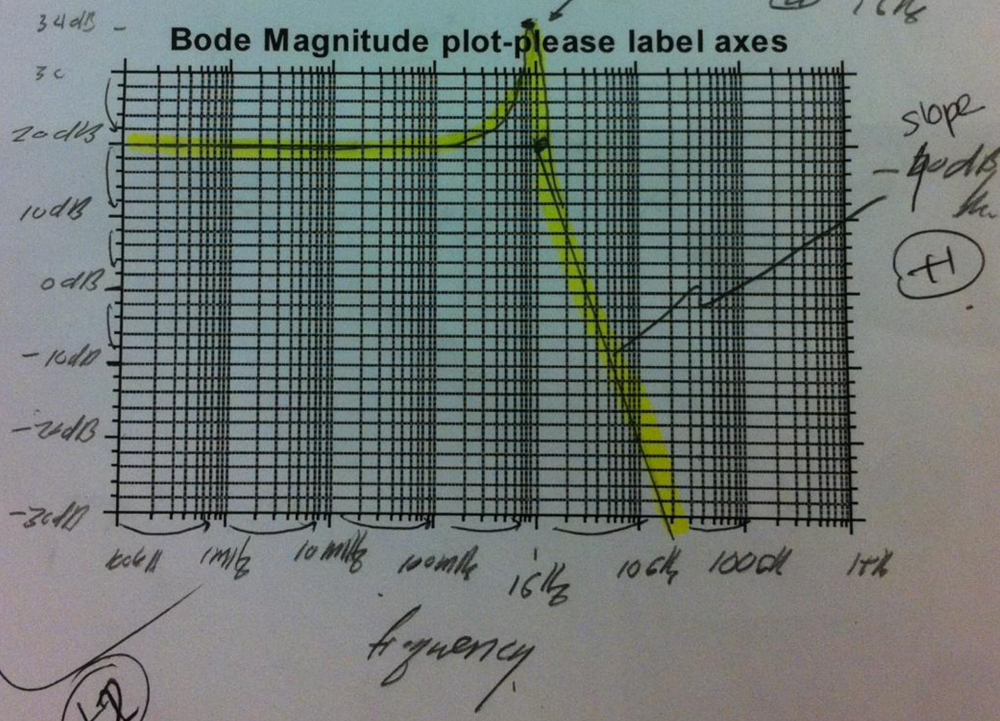
where $\omega_n/2\pi = 1$ GHz , $\zeta = 0.1$, and $H_{DC} = 10$.

Part a, 5 points

Make an **accurate** asymptotic Bode plot of $\| H \|$, labeling and dimensioning axes and clearly labeling all slopes and all critical frequencies.

(+1)

peak at +34 dB @ 1 GHz



(+1)

(+1)

(+2)

$$H(s) = \frac{H_{DC}}{1 + s(2\zeta/\omega_n) + s^2/\omega_n^2} \xrightarrow{s \to j\omega} \frac{H_{DC}}{1 - \frac{\omega^2}{\omega_n^2} + j\frac{\omega}{\omega_n} \cdot 2\zeta}$$

$$= \begin{cases} H_{DC} & \omega \ll \omega_n \\ -j H_{DC} \frac{1}{2\zeta} & \omega = \omega_n \quad \frac{1}{2\zeta} = 5 \\ -H_{DC} \cdot \frac{\omega_n^2}{\omega^2} & \omega \gg \omega_n. \end{cases}$$

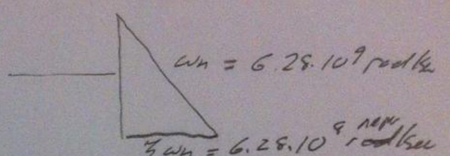
use the asymptotes to make plot.

$20\log_{10} 5 = 14\text{dB}.$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$= 6.28(10^9)$ rad/sec $\cdot\sqrt{1-\zeta^2}$

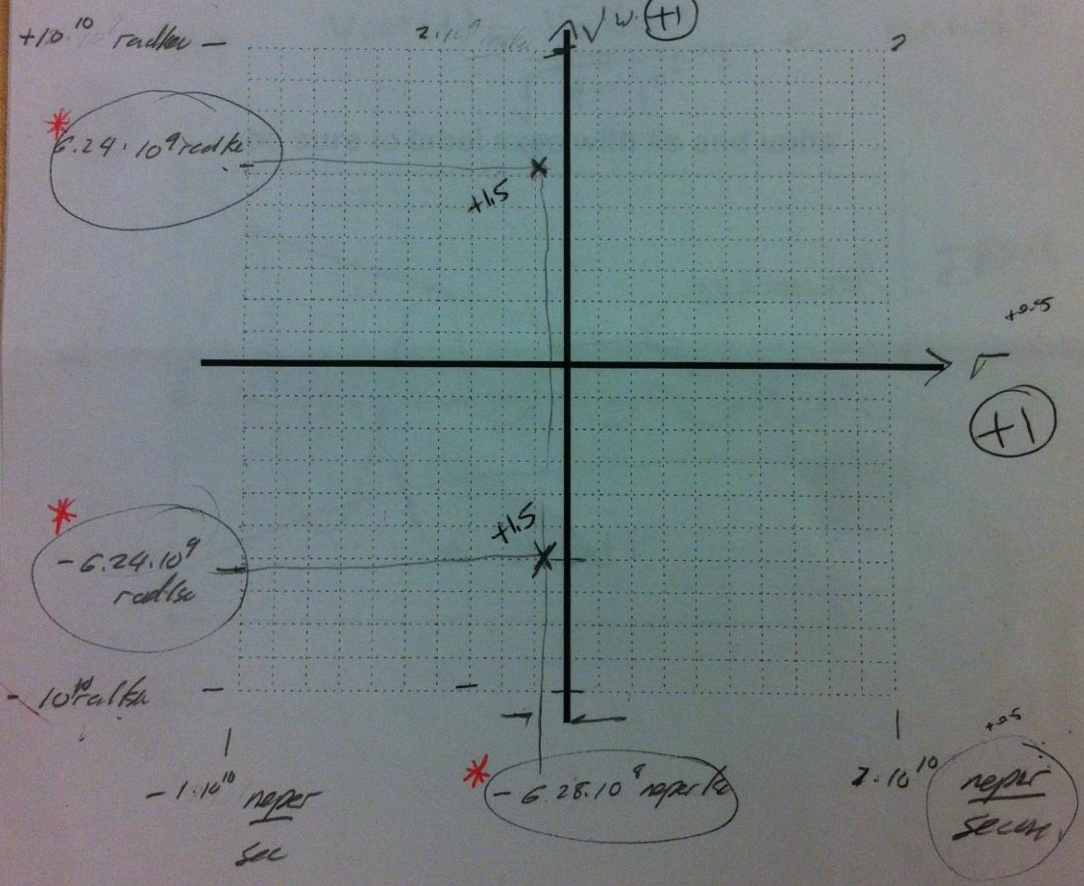
$= 6.24\cdot10^9$ rad/sec



Part b, 5 points

Make an **accurate** root locus plot of H(s), labeling and dimensioning axes and clearly labeling all critical frequencies.

be sure to label axes with #s and units



Part C,10 points

$V_{in}(t)$ is a 1-Volt-magnitude step-function.

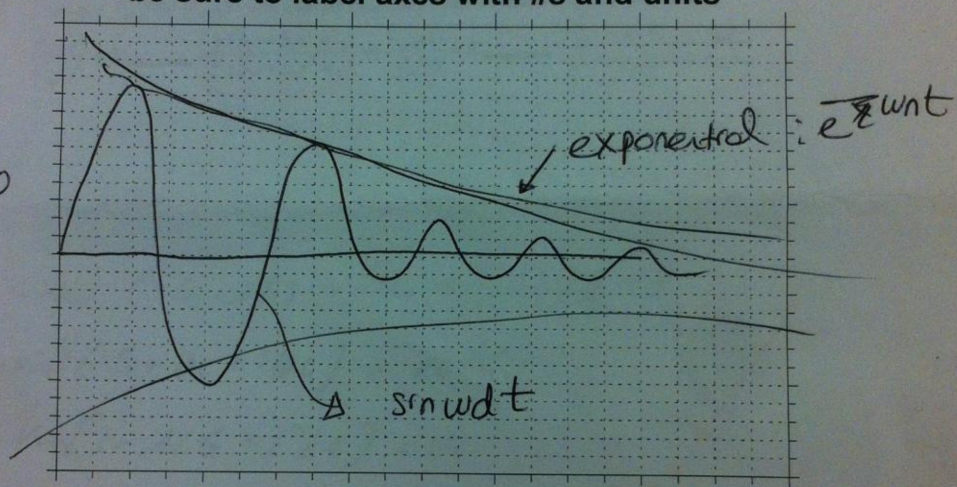
Find $v_{out}(t)$ and make an accurate plot of it below.

+4

$$v_{out}(t) = \frac{V_0 \xi \omega_n}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \sin \omega_d t$$

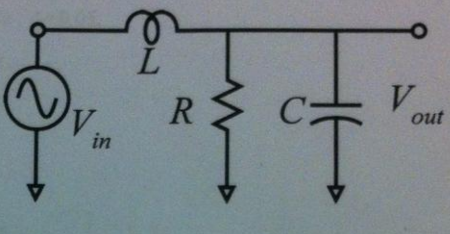
be sure to label axes with #s and units

+6



Problem 7, 15 points

You will be working with the elementary filter/equalizer circuit at right.



Part a, 7 points

Compute $V_{out}(s)/V_{in}(s)$: The answer must be in one of these two standard forms:

$$\frac{V_{out}(s)}{V_{in}(s)} = K_{\text{dimensionless}} \times \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots} \quad \text{or} \quad \frac{V_{out}(s)}{V_{in}(s)} = K_{\text{dimensionless}} \times \frac{(1-s/s_{z1})(1-s/s_{z2})\ldots}{(1-s/s_{p1})(1-s/s_{p2})\ldots}$$

$V_{out}(s)/V_{in}(s)=$ ____________________

$$V_{out}(G+sC) + (V_{out}-V_{in})\cdot 1/sL = 0$$ +3

$$V_{out}[G + sC + 1/sL] = V_{in}/sL$$

$$V_{out}[1 + sLG + s^2LC] = V_{in}$$

form +1

$$\frac{V_{out}}{V_{in}} = H(s) = \frac{1}{1+sLG+s^2LC} = \frac{1}{1+sL/R+s^2LC}$$ +3

$$= \frac{1}{1+s\,2\zeta/(\omega_n)+s^2/\omega_n^2}$$

$\omega_n = \frac{1}{\sqrt{LC}}$: Natural resonant freq.

$\zeta = \frac{1}{2R}\sqrt{\frac{L}{C}}$: damping factor

From $\frac{2\zeta}{\omega_n} = \frac{L}{R}$

Part b, 3 points

Now set $R = 1\ \text{k}\Omega$, $\omega_n/2\pi = 100\ \text{MHz}$, $\zeta = 0.05$,

Find L and C

L= 0.159 μH.

C= 15.9 pF

+2

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}} \quad \text{because } 2\zeta/\omega_n = L/R$$

+1

$$2\zeta/\omega_n = L/R$$

$$\Rightarrow L = \frac{2\zeta}{\omega_n} \cdot R = \frac{2 \cdot 0.05}{2\pi(100\,\text{MHz})} \cdot 1\,\text{k}\Omega$$

$$L = 1.59(10^{-7})\ H = 0.159\,\mu H$$

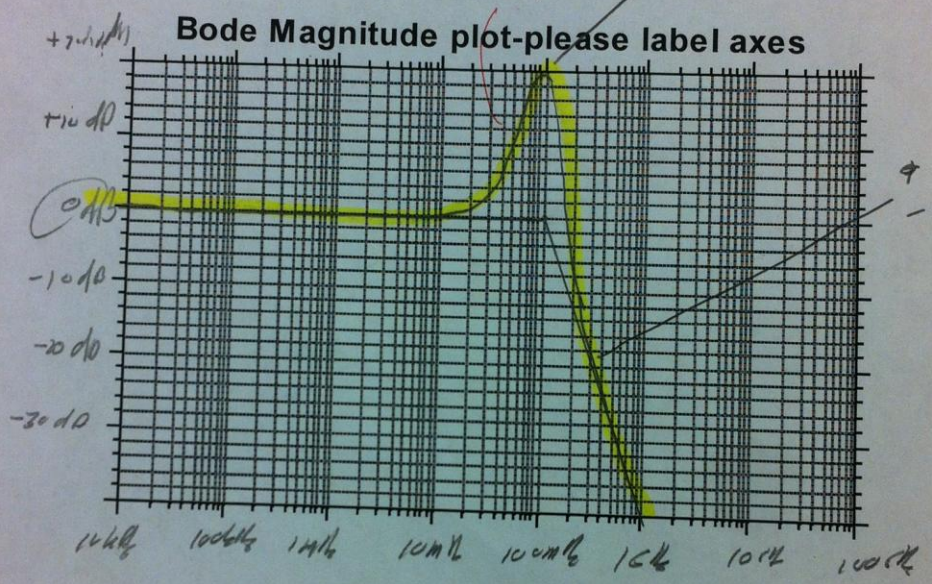
$$\omega_n = \frac{1}{\sqrt{LC}} \rightarrow \omega_n^2 = \frac{1}{LC} \rightarrow C = \frac{1}{L\,\omega_n^2}$$

$$C = \frac{1}{(2\pi \cdot 100\,\text{MHz})^2 \cdot L} = 15.9\,\text{pF}$$

Part c, 5 points

Make an **accurate** asymptotic Bode plot of $\| H \|$, labeling and dimensioning axes and clearly labeling all slopes and all critical frequencies.

20 dBs peak @ 100M



$$H(s) = \frac{1}{1 + s(2\zeta/\omega_n) + s^2/\omega_n^2} = \frac{1}{1 - \frac{\omega^2}{\omega_n^2} + \frac{j\omega}{\omega_n} 2\zeta}$$

$$= \begin{cases} 1 & \omega \ll \omega_n \\ -j\frac{1}{2\zeta} & \omega = \omega_n \\ \frac{\omega_n^2}{\omega^2} & \omega \gg \omega_n \end{cases}$$

$\times \zeta$

$\rightarrow \frac{1}{2\zeta} = 10 \rightarrow$ 20 dBs peak