Question Paper B

1) a

$V_{RF}(t) = 100\,mV \cos(\omega_I t)\cos(\omega_{LO} t) - 100\,mV \sin(\omega_Q t)\sin(\omega_{LO} t)$

$\omega_I = 2\pi \times 100$

$\omega_Q = 2\pi \times 200$

$\omega_L = 2\pi \times 10^9$

$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

$$V_{RF}(t) = \frac{100mV}{4}\left[\cos(\omega_{LO}-\omega_I)t + \cos(\omega_{LO}+\omega_I)t - \cos(\omega_{LO}-\omega_Q)t + \cos(\omega_{LO}+\omega_Q)t\right]$$

$$= 25mV\left[\cos(\omega_{LO}-\omega_I)t + \cos(\omega_{LO}+\omega_I)t - \cos(\omega_{LO}-\omega_Q)t + \cos(\omega_{LO}+\omega_Q)t\right]$$

Freque	10^9-100	10^9+100	10^9-200	10^9+200
Amp	0.025 V	0.025 V	0.025 V	0.025 V
Phase	90	90	-90	90

1 b

$V_{out} = V_{RF}(t)\ V_{LO}(t) / V_0$

$V_{RF}(t) = V_I(t)\cos(\omega_{LO}t) - V_Q(t)\sin(\omega_{LO}t)$

$V_d(t) = \frac{1}{2}\left[V_I(t)\cos^2(\omega_{LO}t) - V_Q(t)\sin(\omega_{LO}t)\cos(\omega_{LO}t)\right]$

$\cos^2\theta = \frac{1}{2}\left[1+\cos 2\theta\right]$

$\cos\theta\sin\theta = \frac{1}{2}\sin 2\theta$

$\cos\theta\sin\theta = \frac{1}{2}\sin 2\theta$

$V_d(t) = \frac{1}{2}\left[V_I(t)\left(\frac{1+\cos 2\omega_{LO}t}{2}\right) - V_Q(t)\frac{\sin 2\omega_{LO}t}{2}\right]$

$= \frac{V_I(t)}{4}$ After Neglecting Higher Frequency terms

$= 0.025\cos(2\pi\, 100t)$

c) $V_{LO}(t) = \cos\omega_{LO}t + \sin\omega_{LO}t$

$V_{RF}(t) = V_I(t)\cos\omega_{LO}t - V_Q(t)\sin\omega_{LO}t$

$V_D(t) = 0.5\left[V_I(t)\cos^2\omega_{LO}t + V_I(t)\cos\omega_{LO}t\sin\omega_{LO}t - V_Q(t)\sin^2\omega_{LO}t) - V_Q(t)\sin\omega_{LO}t\cos\omega_{LO}t\right.$

$\sin^2\theta = \frac{1}{2}\left[1-\cos 2\theta\right]$

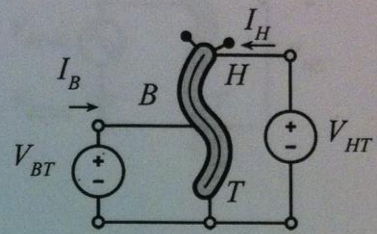
$V_D(t) = \frac{1}{4}\left[V_I(t) - V_Q(t)\right]$ After Neglecting Higher Frequency terms

Problem 2, 30 points

Neurology researchers at UC Santa Cruz have discovered that applying small voltages between the belly (B) and tail (T) of a banana slug results in modulation of the currents between head (H) and tail (T). They find

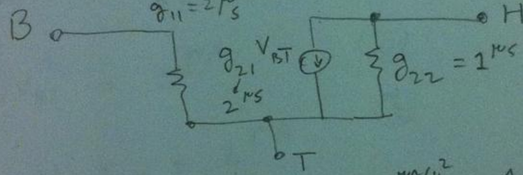
$I_B = K_1 V_{BT}^2$ where $K_1 = 1\mu\text{A/V}^2$ and

$I_H = K_2 V_{BT} V_{HT}$ where $K_2 = 1\mu\text{A/V}^2$



part a, 10 points

Assuming DC voltages $V_{BT,DC}$=1 V and $V_{HT,DC}$=2V, draw below a small-signal equivalent circuit of the slug. Give all small-signal parameters.



4

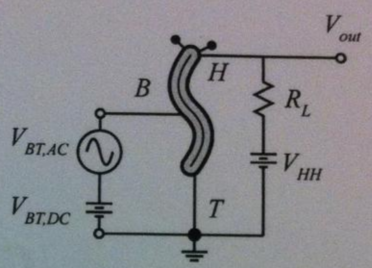
$$g_{11} = \frac{\partial I_B}{\partial V_{BT}} = 2K_1 V_{BT} = 2 \times 1^{\mu A/V^2} \times 1 = \underline{\underline{2\ \mu S}}$$ 2

$$g_{21} = \frac{\partial I_H}{\partial V_{BT}} = K_2 V_{HT} = 1^{\mu A/V^2} \times 2^{V} = \underline{\underline{2\ \mu S}}$$ 2

$$g_{22} = \frac{\partial I_H}{\partial V_{HT}} = K_2 V_{BT} = 1^{\mu A/V^2} \times 1^{V} = \underline{\underline{1\ \mu S}}$$ 2

part b, 10 points

We now set $V_{BT,DC}$ =1 V and R_L=1 MΩ. We want the DC value of V_{out} to be 2.0 Volts. Find the necessary value of V_{HH}. Find the DC currents entering the H and B electrodes.



V_{HH} = 4^{V}

DC current into H electrode = $2\,\mu A$

DC current into B electrode = $1\,\mu A$

$V_{HT} = 2^{V}$ $\quad V_{BT} = 1^{V}$

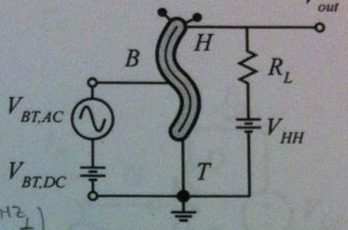
$I_B = K_1 V_{BT}^2 = 1\,\mu A$ 3

$I_H = K_2 V_{BT} V_{HT} = 2\,\mu A$ 3

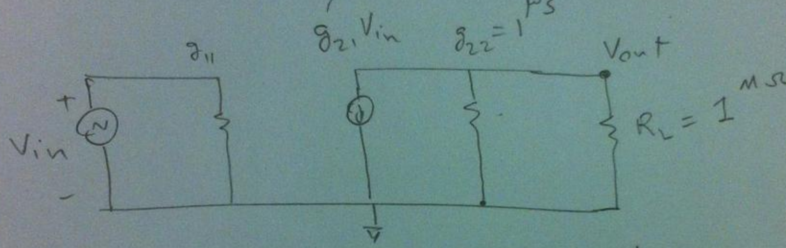
$V_{HH} = V_H + R_L I_H = 2^{V} + 2^{V} = 4^{V}$ 4

part c, 10 points

Continuing with the values you have found in parts (a) and (b), a 1 kHz sine wave of 1 mV RMS amplitude is applied, as shown, to the belly. Find the resulting AC output voltage $V_{out}(t)$



$V_{out}(t) =$ $-1^{mV}\sqrt{2}\,\sin(2\pi \cdot 1^{kHz} t)$

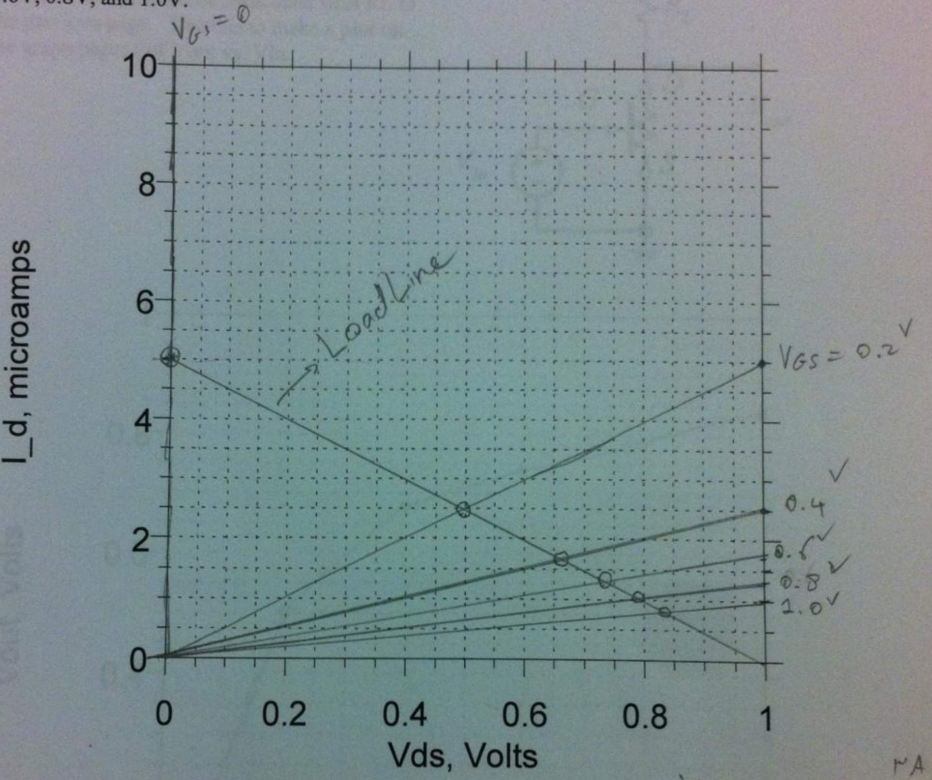


$R_{eq} = 1^{M\Omega} \parallel 1^{M\Omega} = \underline{500^{k\Omega}}$

$V_{out} = -g_{21} V_{in} (500^{k\Omega}) = -V_{in}$

Part a, 5 points

On the graph paper below. First plot I_D as a function of V_{DS} for $V_{gs} = 0V, 0.2V, 0.4V, 0.6V, 0.8V$, and $1.0V$.

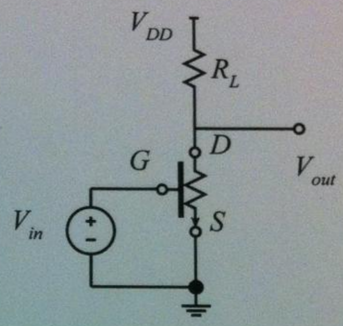


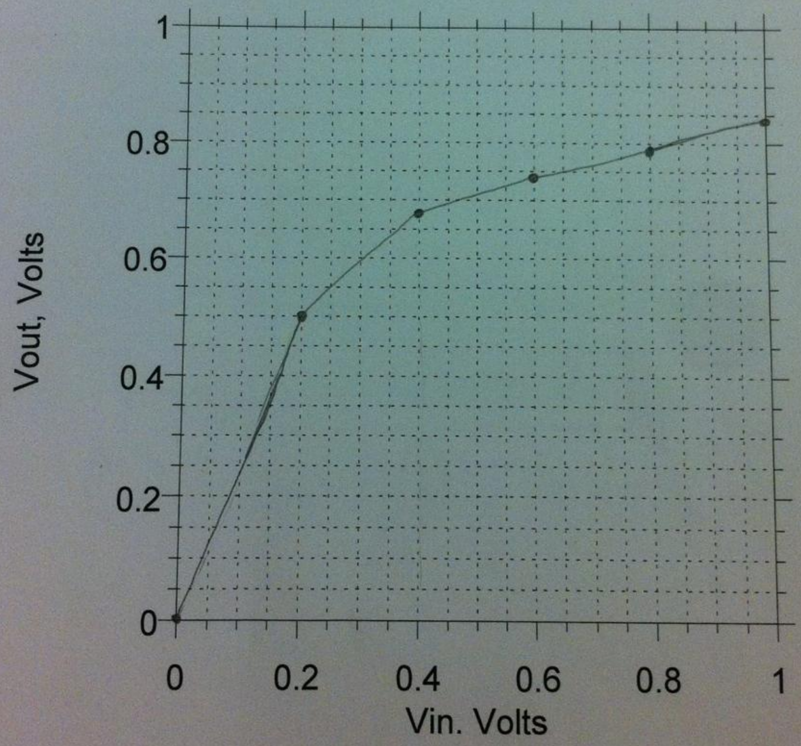
$$I_D = K \frac{V_{DS}}{V_{GS}}, \quad K = 1\ \mu A$$

$I_D =$	V_{GS}
∞	0
$5\ \mu A/V \cdot V_{DS}$	0.2
$2.5\ \mu A/V \cdot V_{DS}$	0.4
$1.66\ \mu A/V \cdot V_{DS}$	0.6
$1.25\ \mu A/V \cdot V_{DS}$	0.8
$1.0\ \mu A/V \cdot V_{DS}$	1.0

Part b, 10 points

We now connect the device, as shown to a V_{DD} =1.0 V power supply and R_L = 200 kΩ load resistor. Add the loadline associated with RL to the previous page. Use this to make a plot on the graph paper (of Vout vs. Vin.





Problem 4, 25 points

Q1 and Q2 are mobility-limited NFETs, i.e.

$I_d = (\mu C_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$

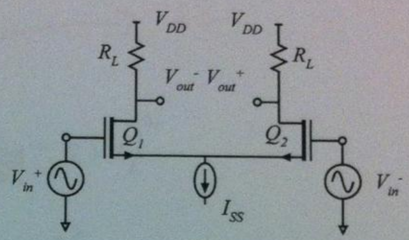
for $V_D > V_g - V_{th}$.

We have

$\lambda = 0\text{V}^{-1}$, $(\mu C_{ox} W_g / 2L_g) = 4\text{mA/V}^2$,

$V_{th} = 0.3\text{V}$, $V_{DD} = 5\text{V}$, $R_L = 5\text{k}\Omega$ and

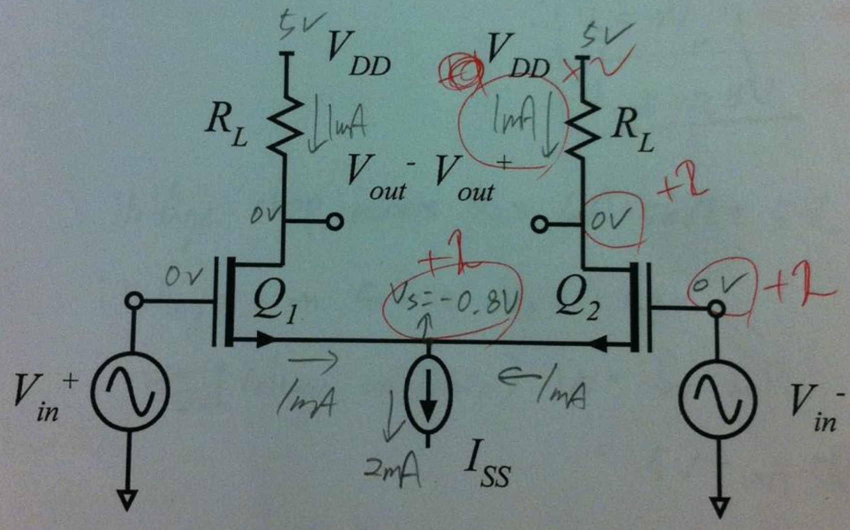
I_{ss}=2 mA.



<u>part a, 10 points</u>

Under DC bias conditions, the input voltages are both set to zero.

The DC voltages on all nodes and the DC currents through all conductors. Indicate these on the circuit diagram below.



$$I_D = \left(4mA/V^2\right)(V_{gs} - 0.3V)^2$$

But $I_D = 1mA$ → +2

$$\rightarrow \quad 1mA = \frac{4mA}{V^2}(V_{gs} - 0.3V)^2$$

$$\frac{1}{4}V^2 = (V_{gs} - 0.3V)^2$$

$$0.5V = \frac{1}{2}V = V_{gs} - 0.3V \Rightarrow V_{gs} = 0.8V$$ +2

$\therefore \boxed{V_g = 0V}$ ← +2

$\boxed{V_s = -0.8V}$ ← +2

Voltage drop across $R_L = 1mA \cdot 5k\Omega = 5V$

Voltage on Source $V_s = -0.8V$

Voltage on Drain) $V_D = V_{DD} - I_D R_L$

$= 5V - 1mA \cdot 5k\Omega = 0V$

+2

10/10

part b, 5 points

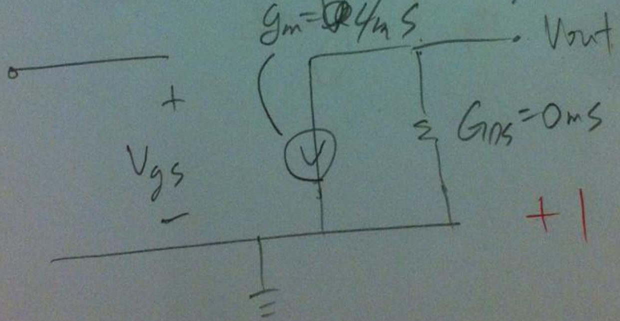
Draw the *FET* small signal equivalent circuit model, giving the values of all small-signal parameters.

$$I_D = (4mA/V^2)(V_{gs}-0.3V)^2$$

$$G_{DS} = g_{22} = \frac{\partial I_D}{\partial V_{DS}} = 0$$ +2

$$g_m = g_{21} = \frac{\partial I_D}{\partial V_{gs}} = \frac{4mA}{V^2} \times 2(V_{gs}-0.3V)\Big|_{V_{gs}=0.8}$$

$$= 4mA/V = 4mS$$ +2

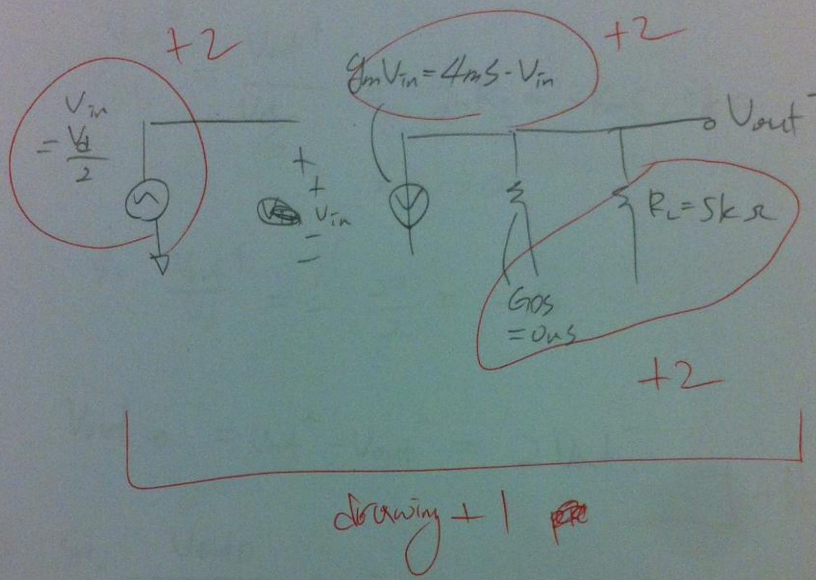


+1

5/5

part c, 7 points

Now, setting $V_{in}^{+} = V_{in,d}/2$ and $V_{in}^{-} = -V_{in,d}/2$, draw a small-signal equivalent circuit of the *amplifier*. Please feel free to exploit the natural symmetry of the circuit to simplify the diagram.



7/7

part d, 8 points

$V_{in,d}$ is a 1 mV (peak) amplitude sine wave at 1 kHz. Find the AC small-signal differential output voltage $V_{out,D}(t) = V_{out}^{+} - V_{out}^{-}$.

$$\frac{V_{out}^{+}}{V_{in}} = \frac{V_{out}^{+}}{V_d/2} = -g_m R_L = -4mS \times 5k\Omega$$

$$= -20$$

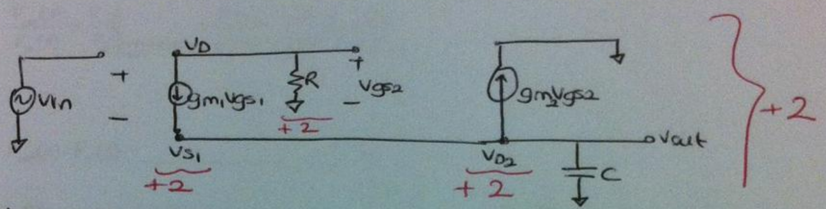
So, $\frac{V_{out}^{+}}{V_d} = -\frac{20}{2} = -10$

$V_{out,D} = V_{out}^{+} - V_{out}^{-} = 2V_{out}^{-}$ +4

So, $\frac{V_{outD}}{V_{in}} = \mp 20\ V/V$ +4

8/8

Part a, 8 points
replacing the transistors with their small-signal models, draw a small-signal equivalent circuit, labeling all elements and all control voltages.



$V_{gs1} = V_{in} - V_{out}$

Part b, 12 points

Compute $V_{out}(s)/V_{in}(s)$: The answer must be in one of these two standard forms:

$$\frac{V_{out}(s)}{V_{in}(s)} = \left.\frac{V_{out}}{V_{in}}\right|_{\substack{\text{low-frequency}\\ \text{value}}} \times \frac{1+b_1 s + b_2 s^2 + \ldots}{1 + a_1 s + a_2 s^2 + \ldots} \quad \text{or}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \left.\frac{V_{out}}{V_{in}}\right|_{\substack{\text{low-frequency}\\ \text{value}}} \times \frac{(1-s/s_{z1})(1-s/s_{z2})\ldots}{(1-s/s_{p1})(1-s/s_{p2})\ldots}$$

$V_{out}(s)/V_{in}(s) =$ ______________________

$\Sigma I = 0$

① $g_{m1} \underbrace{V_{gs1}}_{V_{in}-V_{out}} + \frac{V_D}{R} = 0$ +4

② $g_{m1} \underbrace{V_{gs1}}_{V_{in}-V_{out}} = g_{m2} \underbrace{V_{gs2}}_{V_D} + V_{out}\, sC$ +4

$g_{m1} V_{in} - g_{m1} V_{out} + \frac{V_D}{R} = 0$

$g_{m1} V_{gs1} (1 + R g_{m2}) = V_{out}\, sC$

+2

$\Rightarrow g_{m1} (1 + R g_{m2}) V_{in} = (g_{m1}(1+R g_{m2}) + sC)\, V_{out}$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{g_{m1} + R g_{m1} g_{m2}}{g_{m1} + R g_{m1} g_{m2} + sC}}$$

+2

Part c, 5 points

Now $g_{m1} = g_{m2} = 1$ ms, $R = 1\ \mathrm{k}\Omega$, $C = 1$ pF.

Give the frequencies, in Hz, of any and all poles and zeros. Clearly indicate if any lie in the right half of the S-plane

1st pole frequency=____________, 2nd pole frequency=__0______,
3rd pole frequency=__0______

1st zero frequency=__0______, 2nd zero frequency=__0______,
3rd zero frequency=__0______

$$H(s) = \frac{1}{1 + s\,\frac{1\,\mathrm{pF}}{2\,\mathrm{ms}}} = \frac{1}{1 + \frac{s}{2\times10^9}}$$

$$w_p = -2\times10^9\ \mathrm{rad/sec}$$

$$f_p = \frac{2}{2\pi}\times10^9\ \mathrm{Hz} = \underline{\underline{0.318\ \mathrm{GHz}}}$$

Problem 6, 20 points

We have a circuit for which

Look @ exam A.

It's the same question

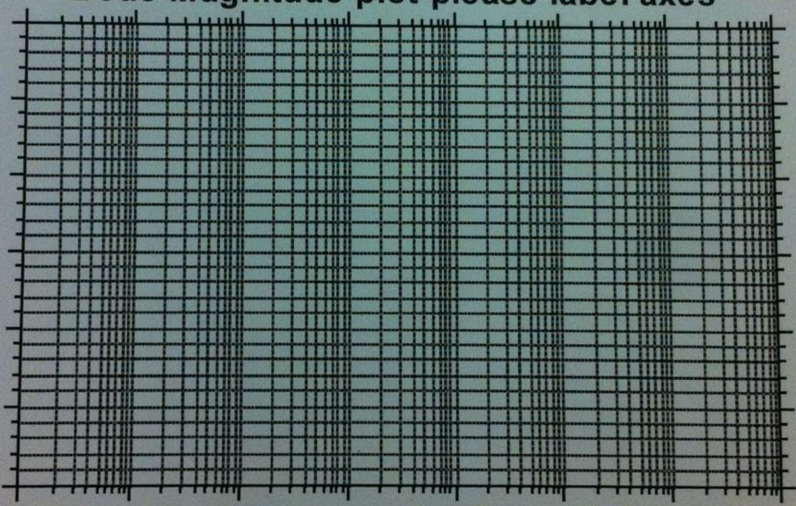
$$\frac{V_{out}(s)}{V_{in}(s)} = H(s) = H_{DC} \times \frac{1}{1 + s(2\zeta/\omega_n) + s^2/\omega_n^2}.$$

where $\omega_n/2\pi$ = 1 GHz, ζ =0.1, and H_{DC} =10.

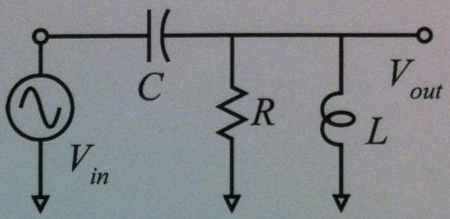
<u>Part a, 5 points</u>

Make an **accurate** asymptotic Bode plot of $\| H \|$, labeling and dimensioning axes and clearly labeling all slopes and all critical frequencies.

Bode Magnitude plot-please label axes



Problem 7, 15 points
You will be working with the elementary filter/equalizer circuit at right.



Part a, 7 points

Compute $V_{out}(s)/V_{in}(s)$: The answer must be in one of these two standard forms:

$$\frac{V_{out}(s)}{V_{in}(s)} = K_{\text{dimensiuoless}} \times \frac{1+b_1 s+b_2 s^2+\ldots}{1+a_1 s+a_2 s^2+\ldots} \text{ or } \frac{V_{out}(s)}{V_{in}(s)} = K_{\text{dimensionless}} \times \frac{(1-s/s_{z1})(1-s/s_{z2})\ldots}{(1-s/s_{p1})(1-s/s_{p2})\ldots}$$

$V_{out}(s)/V_{in}(s) =$ ______________________________

$$V_{out}\left(G+\frac{1}{sL}\right)+(V_{out}-V_{in})sC=0 \quad +3$$

$$V_{out}\left(G+\frac{1}{sL}+sC\right)=V_{in}\cdot sC$$

$$V_{out}\left(1+\frac{1}{s^2LC}+\frac{G}{sC}\right)=V_{in} \quad ; G=\frac{1}{R}$$

answer +3

$$V_{out}/V_{in}=\frac{1}{\left(1+\frac{1}{s^2LC}+\frac{1}{sRC}\right)}=\left[\frac{LCs^2}{\boxed{1+\frac{L}{R}s+LCs^2}}\right] \text{ form } +1$$

$$=\frac{s^2/\omega_n^2}{1+s\left(\frac{2\zeta}{\omega_n}\right)+s^2/\omega_n^2}$$

$\omega_n=\frac{1}{\sqrt{LC}}$; Natural resonant freq.

$\zeta=\frac{1}{2R}\sqrt{\frac{L}{C}}$: damping factor

Part b, 3 points

Now set $R = 1\ \text{k}\Omega$, $\omega_n/2\pi = 100\ \text{MHz}$, $\zeta = 0.05$,

Find L and C

L= 0.159 μH → +1

C= 15.9 pF → +1

$$\boxed{\begin{aligned} \omega_n &= \frac{1}{\sqrt{LC}} \\ \xi &= \frac{R}{2}\sqrt{\frac{C}{L}} \end{aligned}} \rightarrow +1$$

because $2\xi/\omega_n = \frac{L}{R}$

$$2\xi/\omega_n = L/R$$

$$\Rightarrow L = \frac{2\xi}{\omega_n} \cdot R = \frac{2 \times 0.05 \times 1\text{k}\Omega}{2\pi(100\,\text{MHz})} = 1.59 \times 10^{-7} H$$

$$= 0.159\,\mu H.$$

$$\omega_n = \frac{1}{\sqrt{LC}} \rightarrow \omega_n^2 = \frac{1}{LC} \Rightarrow C = \frac{1}{L\omega_n^2}$$

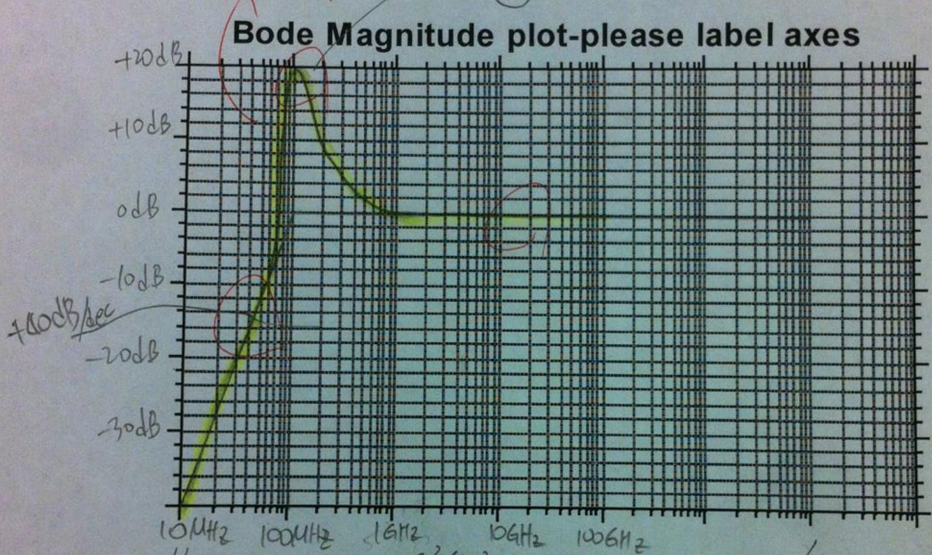
$$C = \frac{1}{(2\pi \times 100\,\text{MHz})^2 \times L} = 1.59 \times 10^{-11} F$$

$$= 15.9 \times 10^{-12} F = 15.9\,pF$$

Part c, 5 points

Make an **accurate** asymptotic Bode plot of $\| H \|$, labeling and dimensioning axes and clearly labeling all slopes and all critical frequencies.

+4 - 20dB peak @ 100MHz.



$$H(s) = \frac{s^2/\omega_n^2}{1+s\left(2\xi/\omega_n\right)+s^2/\omega_n^2} \Rightarrow \frac{1}{\frac{\omega_n^2}{s^2}+\frac{\omega_n}{s}(2\xi)+1}$$

$$= \frac{1}{-\frac{\omega_n^2}{\omega^2}-j\frac{\omega_n}{\omega}(2\xi)+1}$$

$$= \begin{cases} -\frac{\omega^2}{\omega_n^2} & \omega \ll \omega_n \\ \frac{1}{2\xi} & \omega = \omega_n \rightarrow \frac{1}{2\xi} = 10 \rightarrow \text{20dB peak} \\ 1 & \omega \gg \omega_n \end{cases}$$

+1