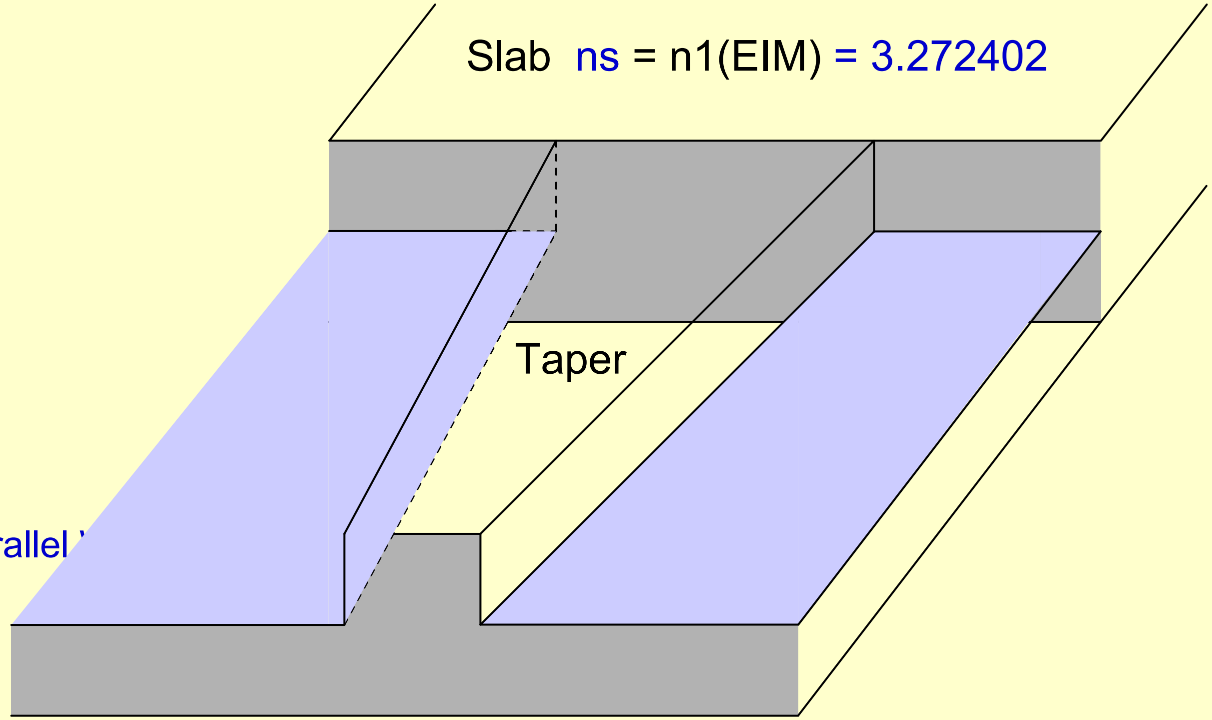
Si-Rib AWG : 40ch-100GHz

λ_0 = 1.55 μm
2a = W = 1.0 μm
H = 0.5 μm
h = 0.25 μm (= *t*)
n1(EIM) = 3.2724
n0(EIM) = 2.9377
Δ_{eff} = 9.7 %
nc = 3.2003 (Semi-Vector FEM)
Nc = 3.7796 (Semi Vector FEM)
R_{min} = 400 μm
Ofst = 0.0 μm
Slit = 0.2 μm
Cut = 0.0 μm
Sprtn = 3.0 μm: Minimum separation of parallel



Sghz = 100 GHz
Vnrghz = 10 GHz
Nfdm = 64
Nscr = 290
Ry = 518.635 μm
ΔL = 12.108 μm
m = 25
Tinc = 100: Coefficient of taper inclination

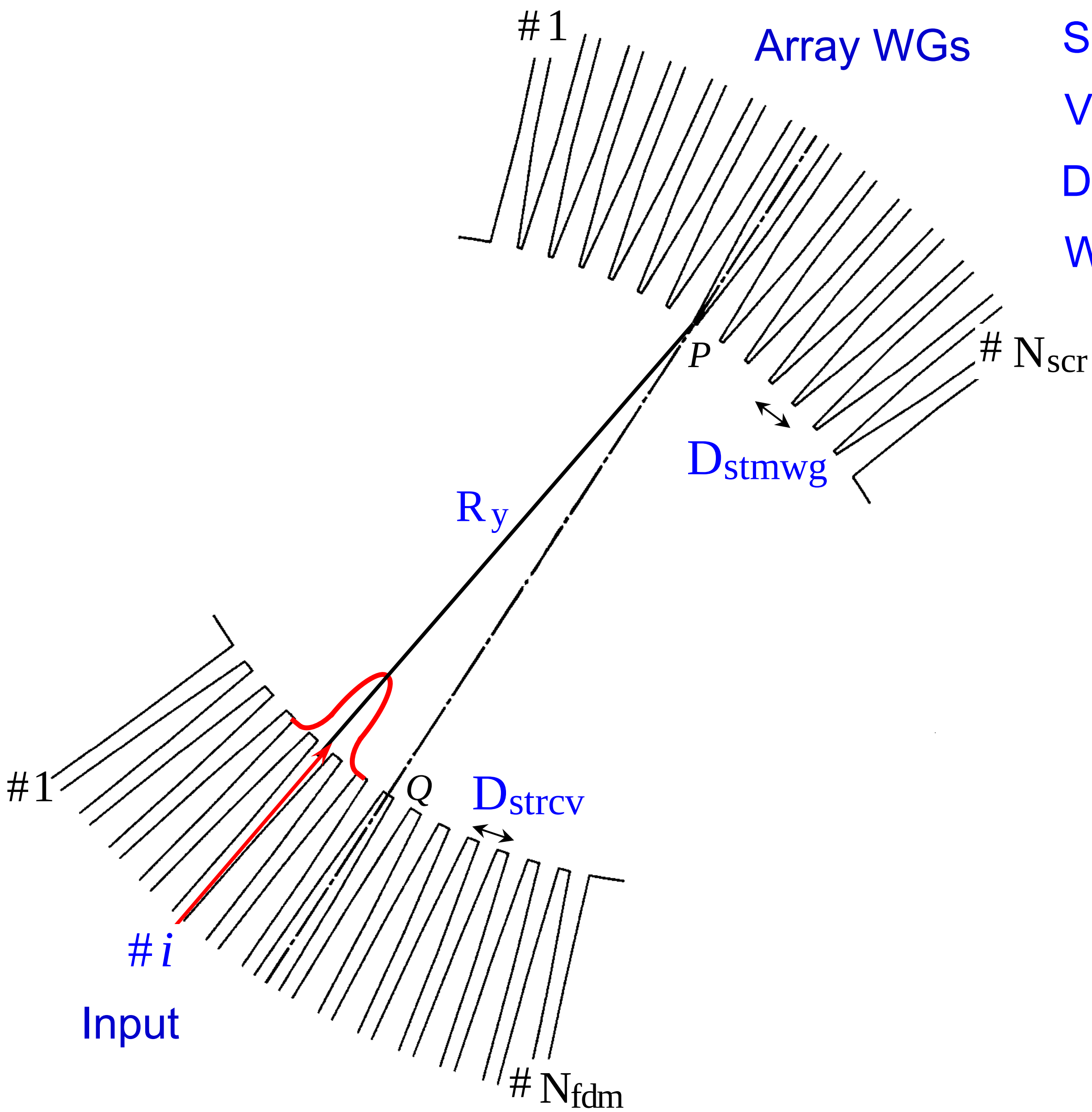
$$Y_{taper} = \left(W_{array} - 2a\right) \cdot T_{inc}/2 = 15\,\mu m$$

Dstmwg = 1.5 μm
(Warray = Dstmwg – (Slit + 2•Cut) = 1.3 μm, Ytaper: Taper WG length at the slab-array interface)
Dstrcv = 2.5 μm
(Diaslp = 1.5 μm, Yslpe: Taper WG length at the I/O-slab interface)

$$Y_{slpe} = \left(D_{iaslp} - 2a\right) \cdot T_{inc}/2 = 25\,\mu m$$

Sfbr = 100 μm: WG spacing at the chip facets

Definition of AWG Parameters



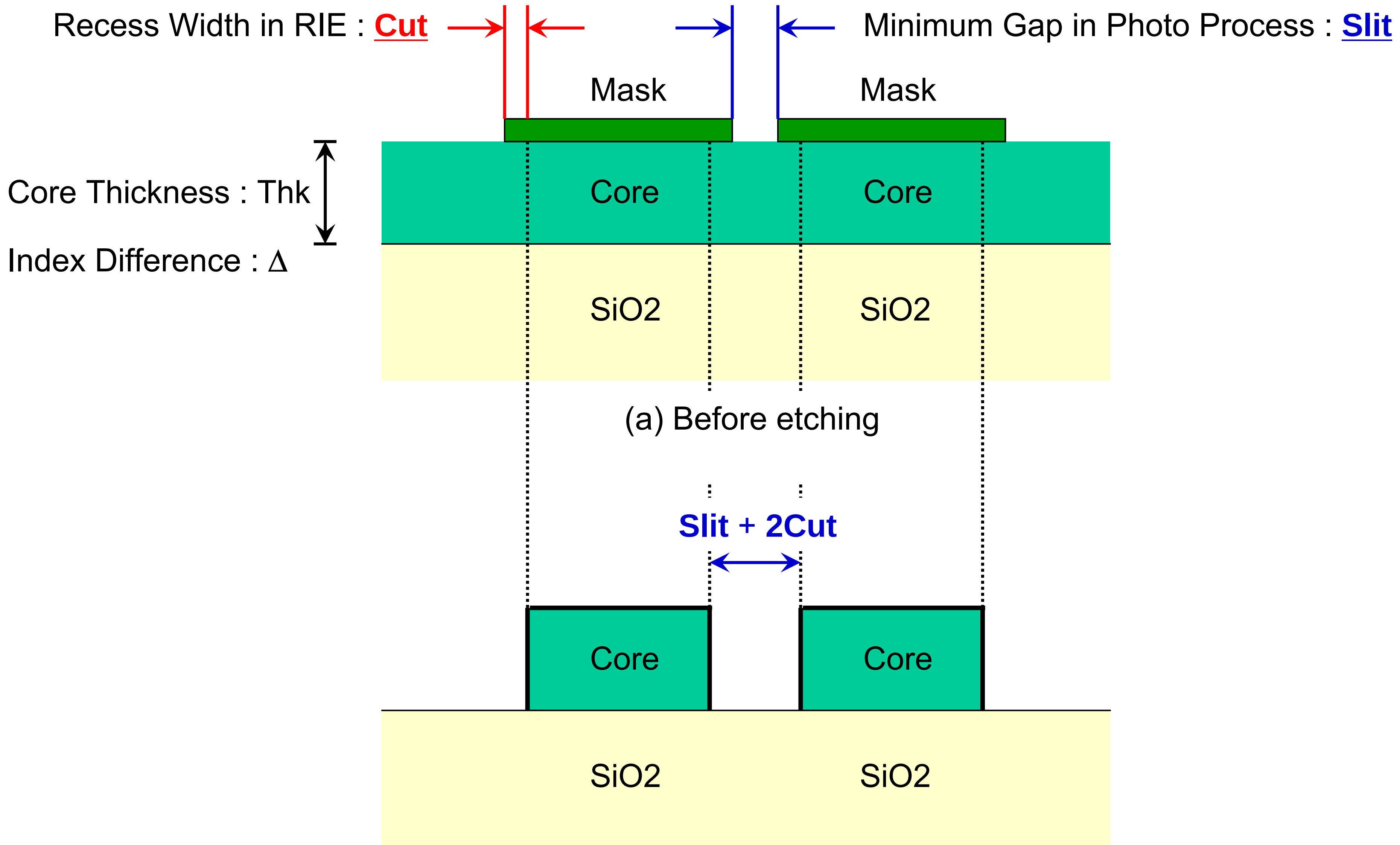
S_{ghz} : Channel spacing (GHz)

V_{nrghz} : Vernier frequency at input (GHz)

D_{iaslp} : Tapered-core width of input WG

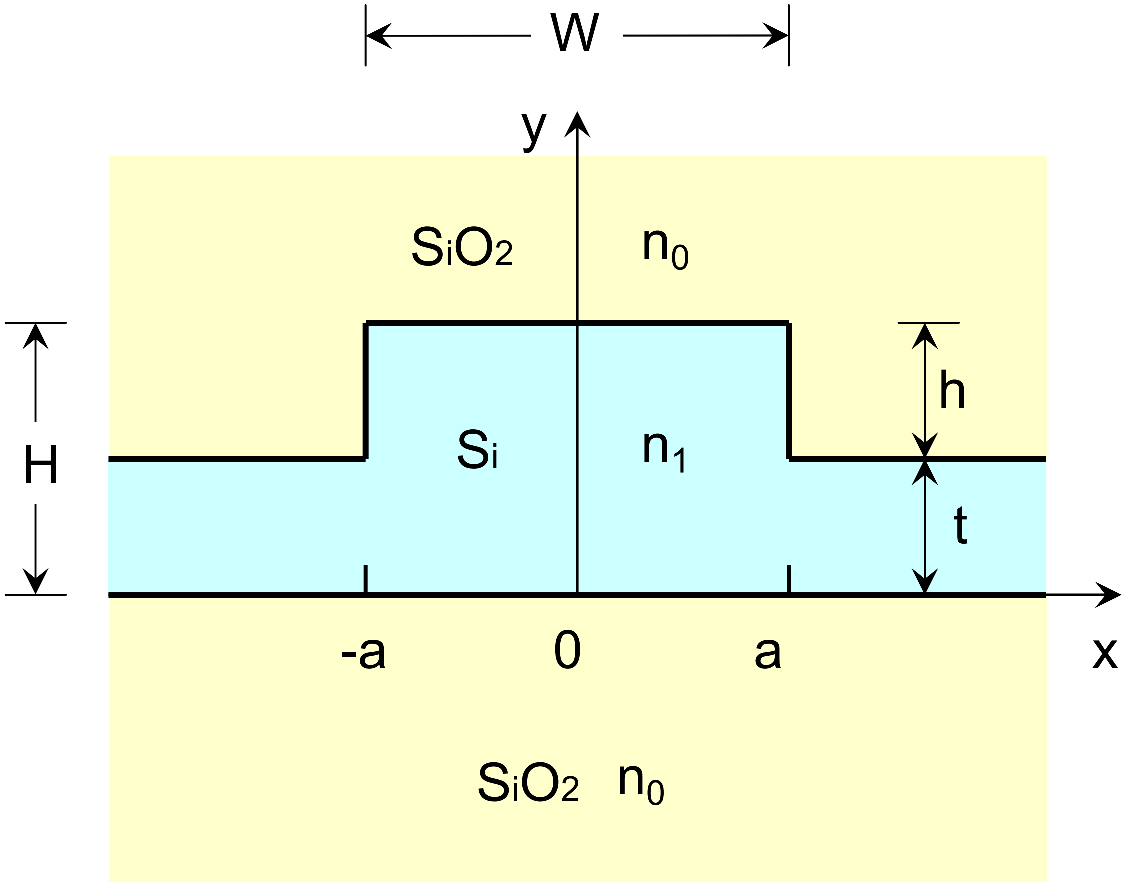
W_{array} : Tapered-core width of array WG

λ_0 = 1.55 μm
Nfdm = 64
Nscr = 290
Sghz = 100 GHZ
Vnrghz = 10 GHz
Ry = 518.635 μm
Dstrcv = 2.5 μm
Dstmwg = 1.5 μm
ΔL = 12.108 μm
m = 25



(a) Before etching
(b) After etching

Si-Rib AWG : 40ch-100GHz



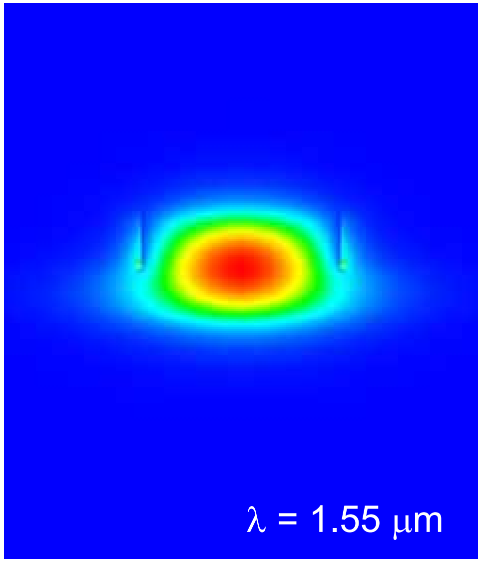
(a) Configuration of rib-type waveguide

Electric Field Distribution

n1(EIM) = 3.2724 ~ 0.5 % error
n0(EIM) = 2.9377 → nc = 3.2168 (EIM)

$$n_c = 3.2003$$

$$N_c = n_c - \lambda \frac{dn_c}{d\lambda} = 3.7796$$

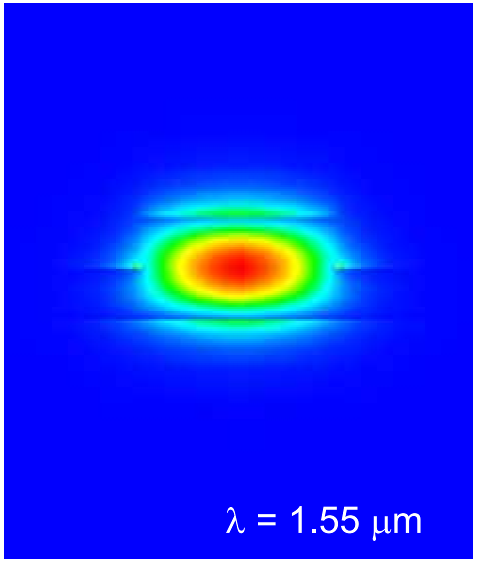


(a) Electric field for TE mode

n1(EIM) = 3.1547 ~ 4.4 % error
n0(EIM) = 2.2951 → nc = 3.2218 (EIM)

$$n_c = 3.0864$$

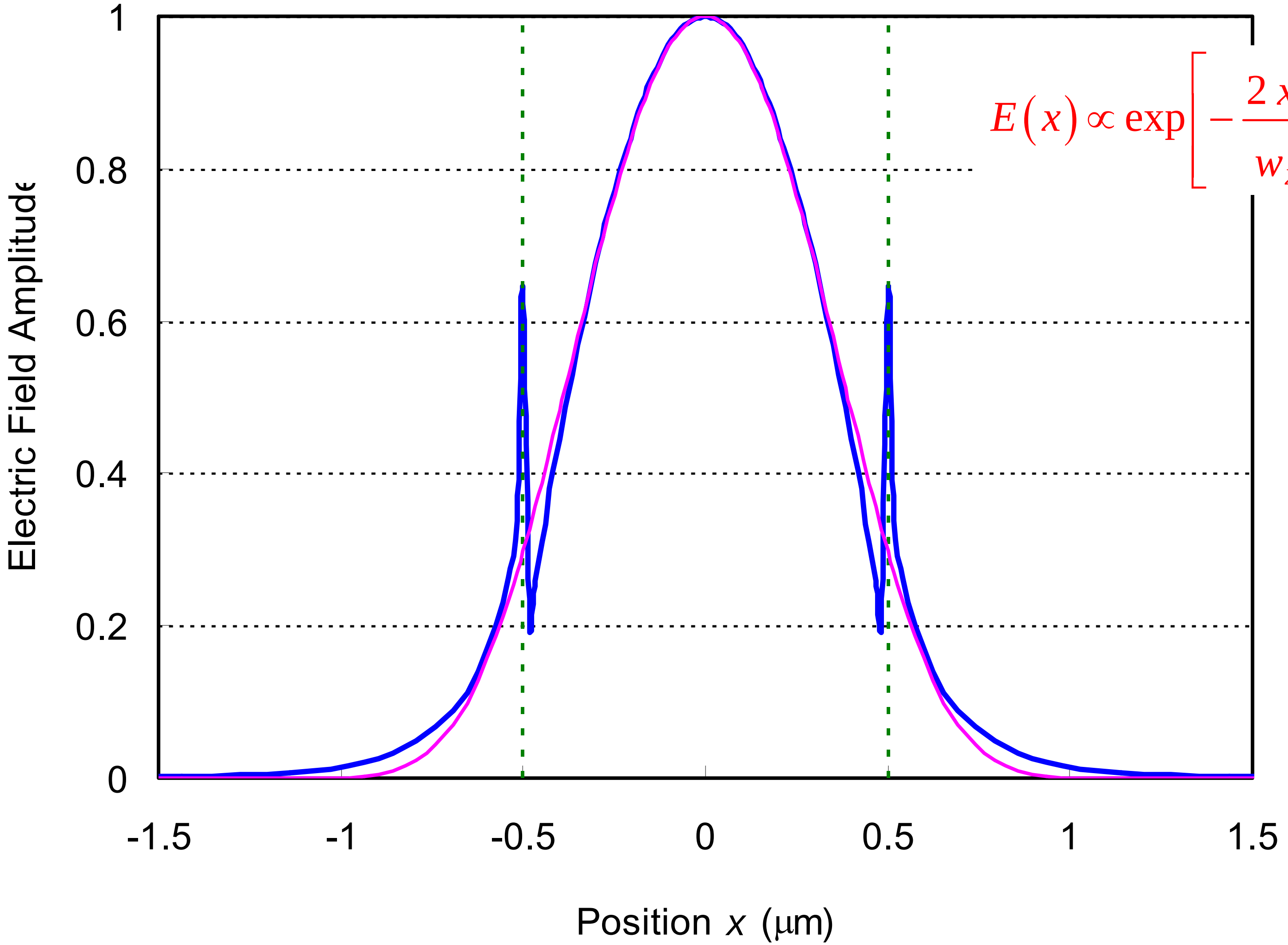
$$N_c = n_c - \lambda \frac{dn_c}{d\lambda} = 3.9794$$



(b) Electric field for TM mode

Electric Field Distribution

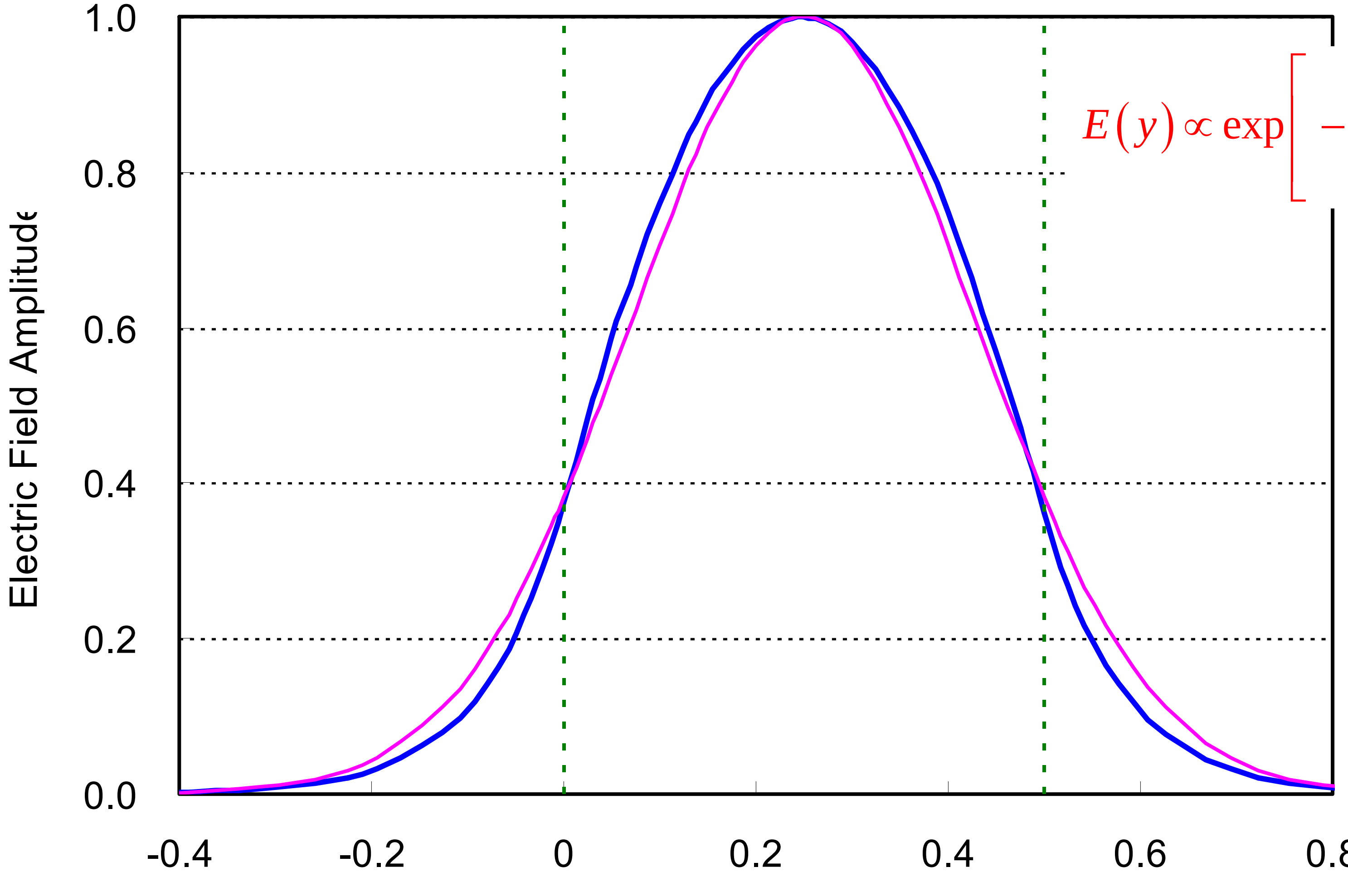
w_{x1} = 0.7 μm %, w_{x2} = 0.9 μm



$$E(x) \propto \exp\left[-\frac{2x^2}{w_{x1}^2} - \frac{2x^4}{w_{x2}^4}\right]$$

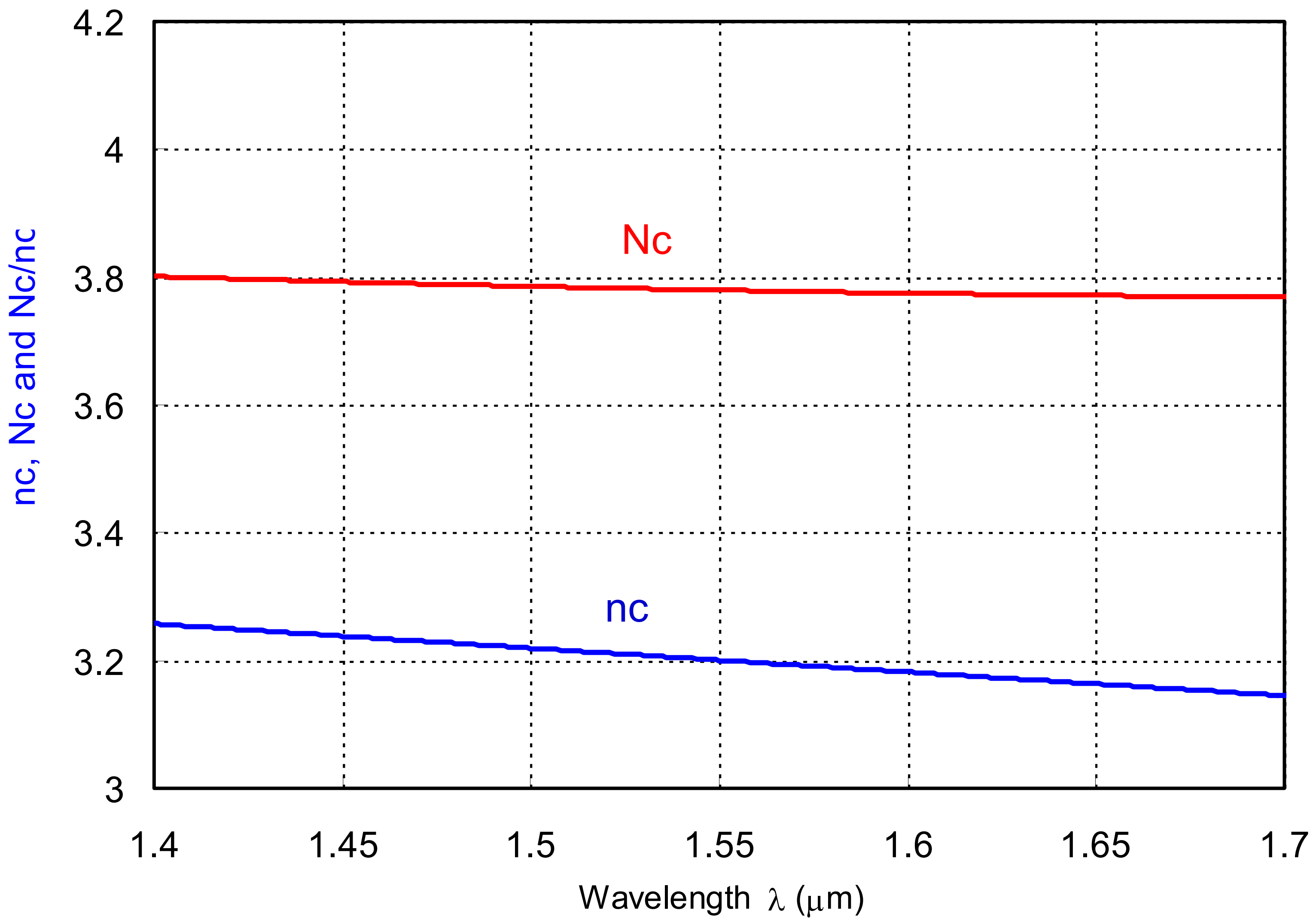
Electric Field Distribution

t = 0.25 μm, w_y = 0.36 μm

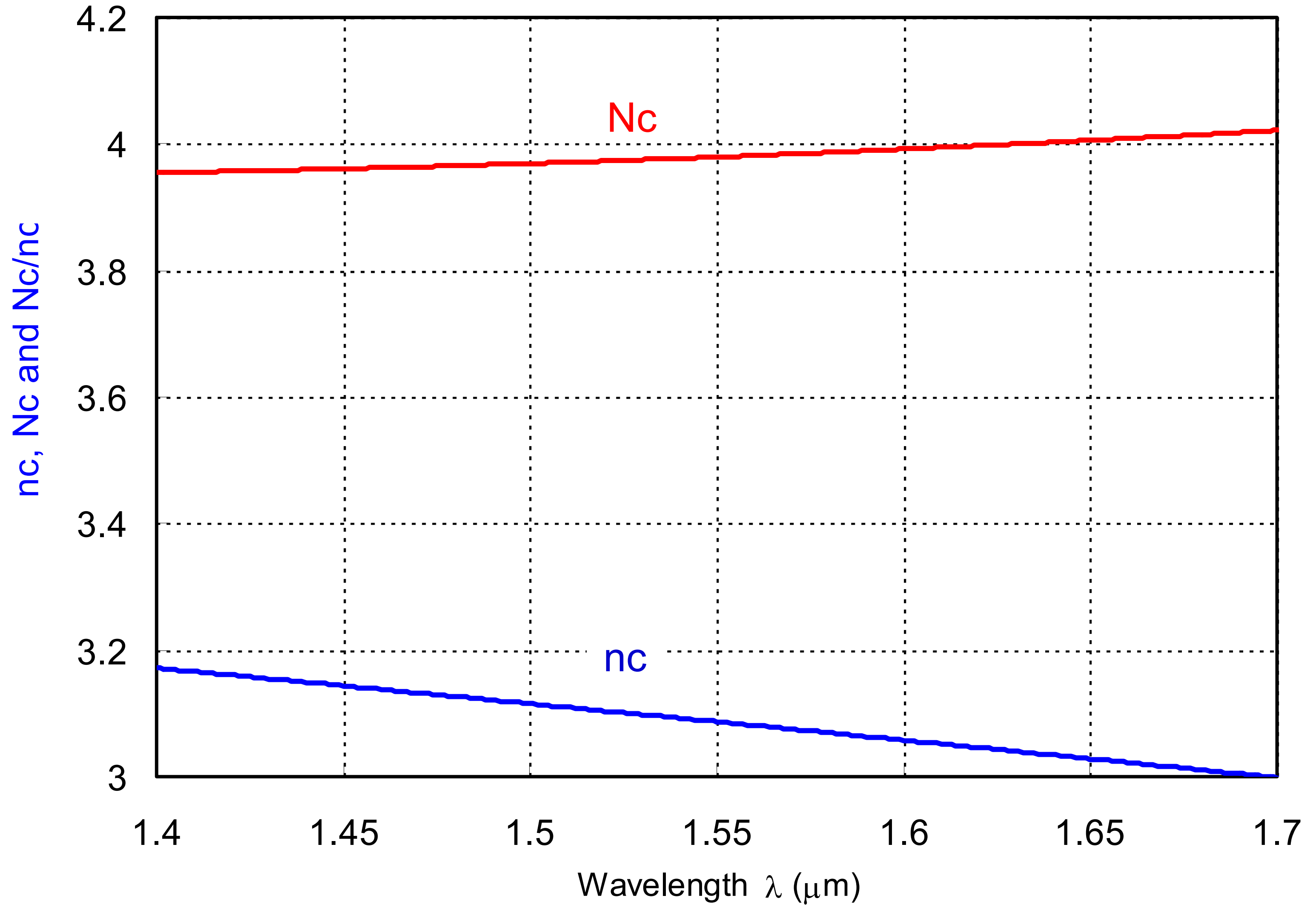


$$E(y) \propto \exp\left[-\frac{2(y-t)^2}{w_y^2}\right]$$

Eigen Value for Ex^{11} Mode



Eigen Value for Ey^{11} Mode



FEM Analysis of Parallel Waveguides

$$\kappa = \frac{(\beta_{even} - \beta_{odd})}{2} = \frac{(n_{even} - n_{odd})\pi}{\lambda}$$

$$P_b(z) = \sin^2(\kappa z)$$

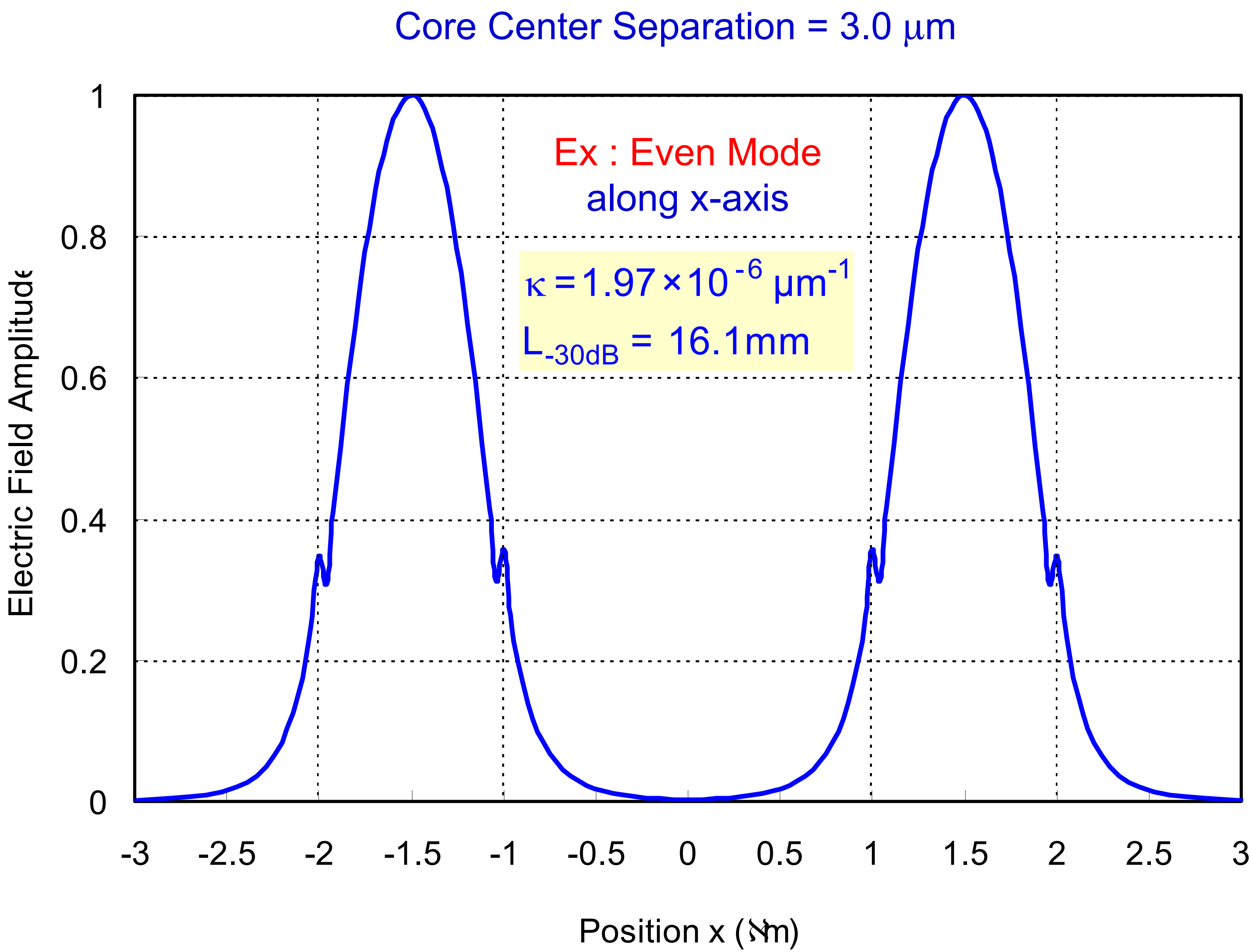
$$L_{100\%} = L_{0dB} = \frac{\pi}{2\kappa} = \frac{\lambda}{2(n_{even} - n_{odd})}$$

$$L_{50\%} = L_{-3dB} = \frac{\pi}{4\kappa} = \frac{\lambda}{4(n_{even} - n_{odd})}$$

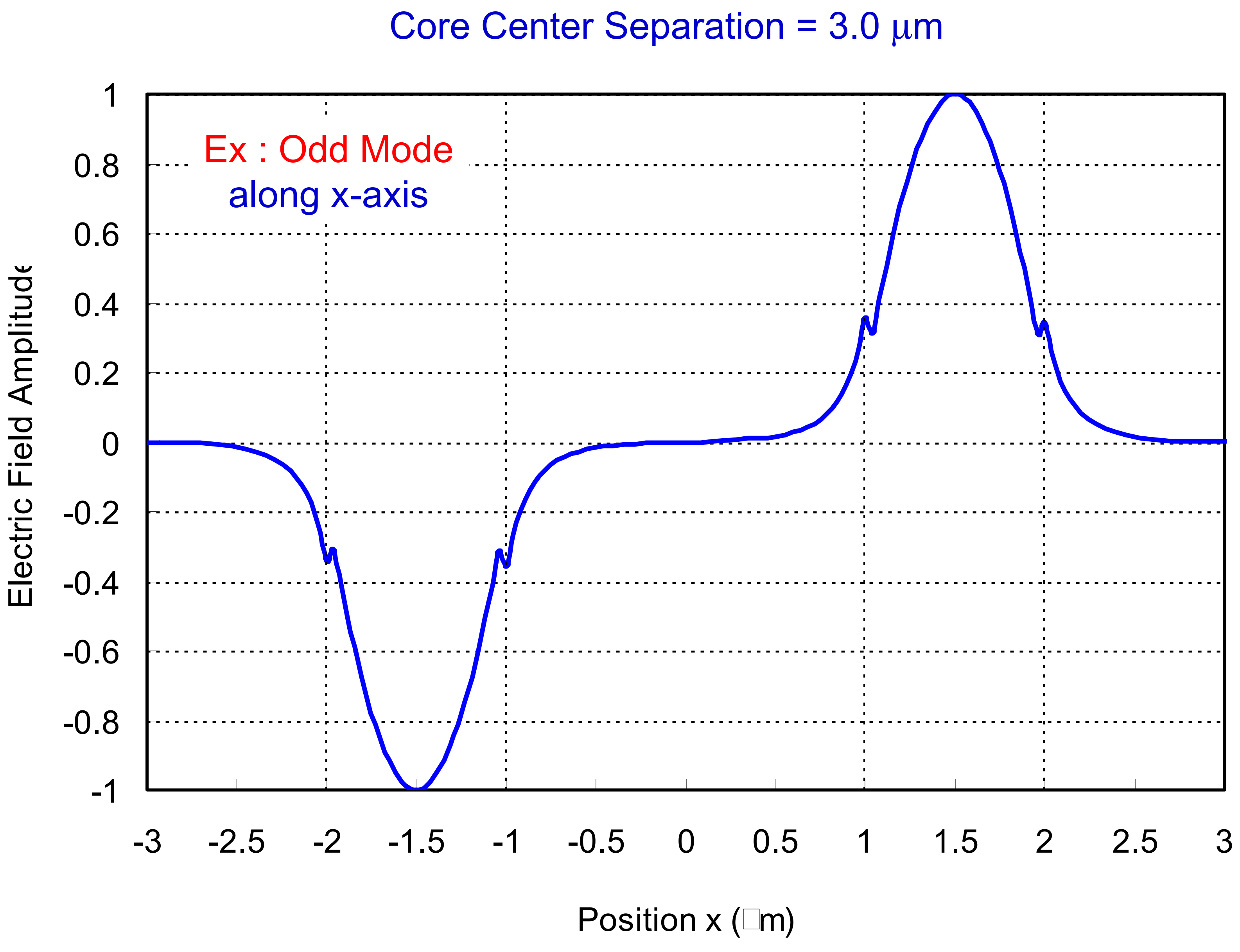
$$ER = -30\text{ dB}$$

$$L_{-30dB} = \frac{1}{\kappa}\sin^{-1}\left\{\exp\left[\frac{ER \cdot \ell n(10)}{20}\right]\right\} = \frac{\lambda}{(n_{even} - n_{odd})\pi}\sin^{-1}\left\{\exp\left[\frac{ER \cdot \ell n(10)}{20}\right]\right\}$$

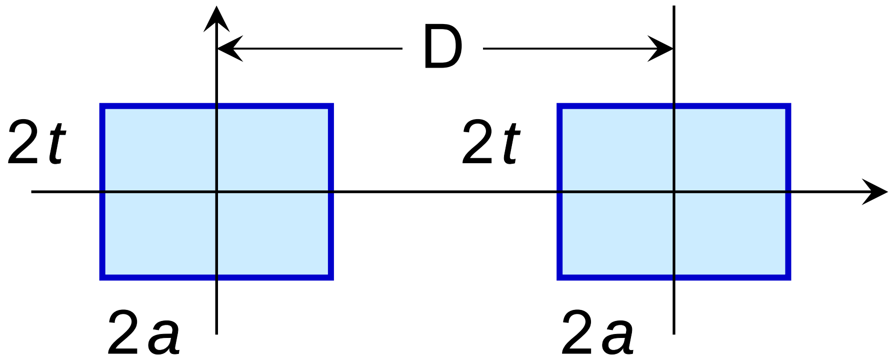
FEM Analysis of Parallel Waveguides



FEM Analysis of Parallel Waveguides



The Minimum Core Center Separation



$$\kappa = \frac{\sqrt{2\Delta}}{a} \frac{u^2 w^2}{(1+w)v^3} \exp\left[-w\left(\frac{D}{a}-2\right)\right]$$

Si-Rib WG by EIM

a = 0.5 μm

Δ = 0.097

λ_0 = 1.55 μm

v = 2.921980

b = 0.826505

D = 3.0 μm

$$\kappa = 2.45 \times 10^{-6}\ \mu m^{-1}$$

CT = -30 dB @ L = 12.9 mm

⇨

Si-Rib WG by FEM

D = 3.0 μm ⇒ Sprtn

$$\kappa = 1.97 \times 10^{-6}\ \mu m^{-1}$$

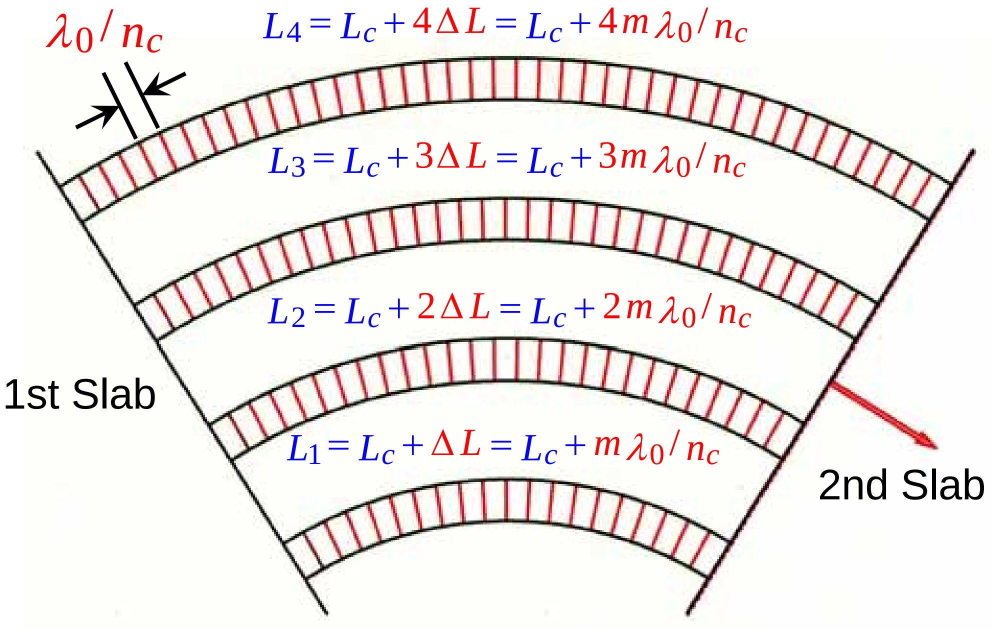
CT = -30 dB @ L = 16.1 mm

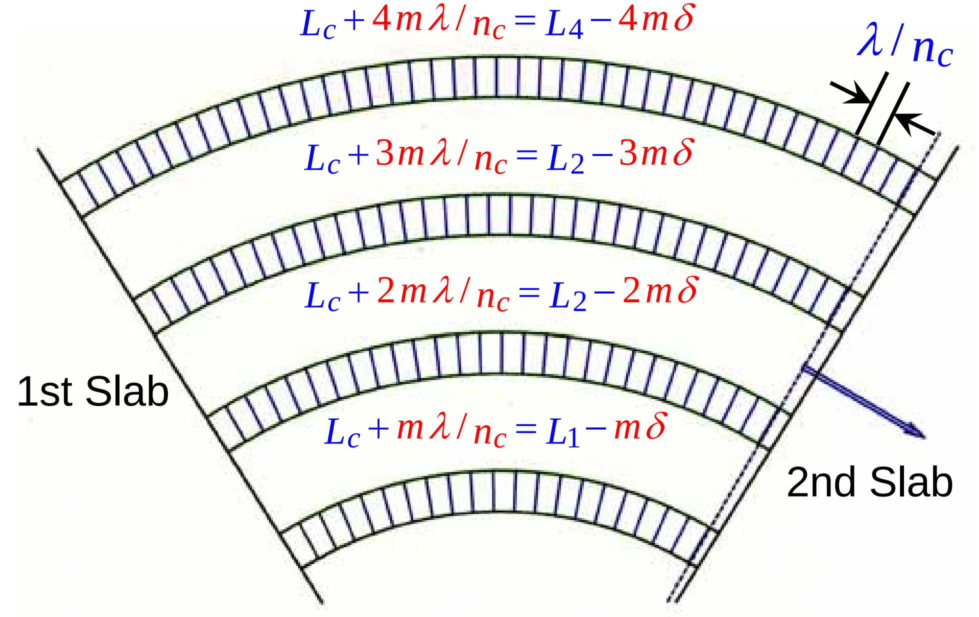
Phase Fronts in Array Waveguides

$\lambda = \lambda_0$

$$\Delta L = m\frac{\lambda_0}{n_c} \Rightarrow \lambda_0 = \frac{n_c \Delta L}{m}$$

$\lambda < \lambda_0$





$$\delta = (\lambda_0 - \lambda)/n_c$$

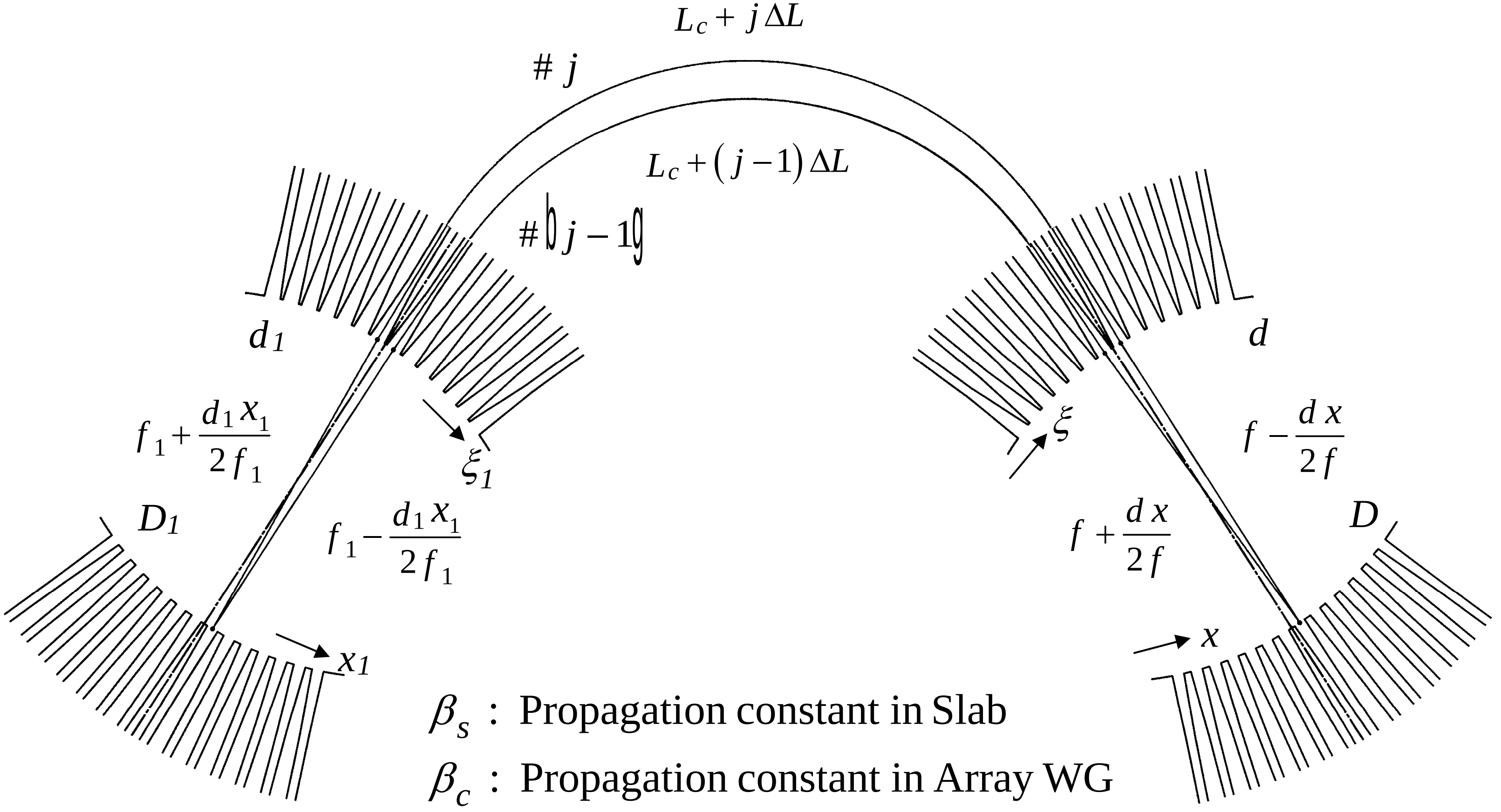
(a) Phase relation for $\lambda = \lambda_0$

(b) Phase relation for $\lambda < \lambda_0$

Interference Condition of AWG

$$\beta_s(\lambda)\left(f_1+\frac{d_1x_1}{2f_1}\right)+\beta_c(\lambda)\left[L_c+j\Delta L\right]+\beta_s(\lambda)\left(f-\frac{dx}{2f}\right)=\beta_s(\lambda)\left(f_1-\frac{d_1x_1}{2f_1}\right)+\beta_c(\lambda)\left[L_c+(j-1)\Delta L\right]+\beta_s(\lambda)\left(f+\frac{dx}{2f}\right)+2m\pi$$

Array WG # *j* — Array WG # (*j* - *1*)

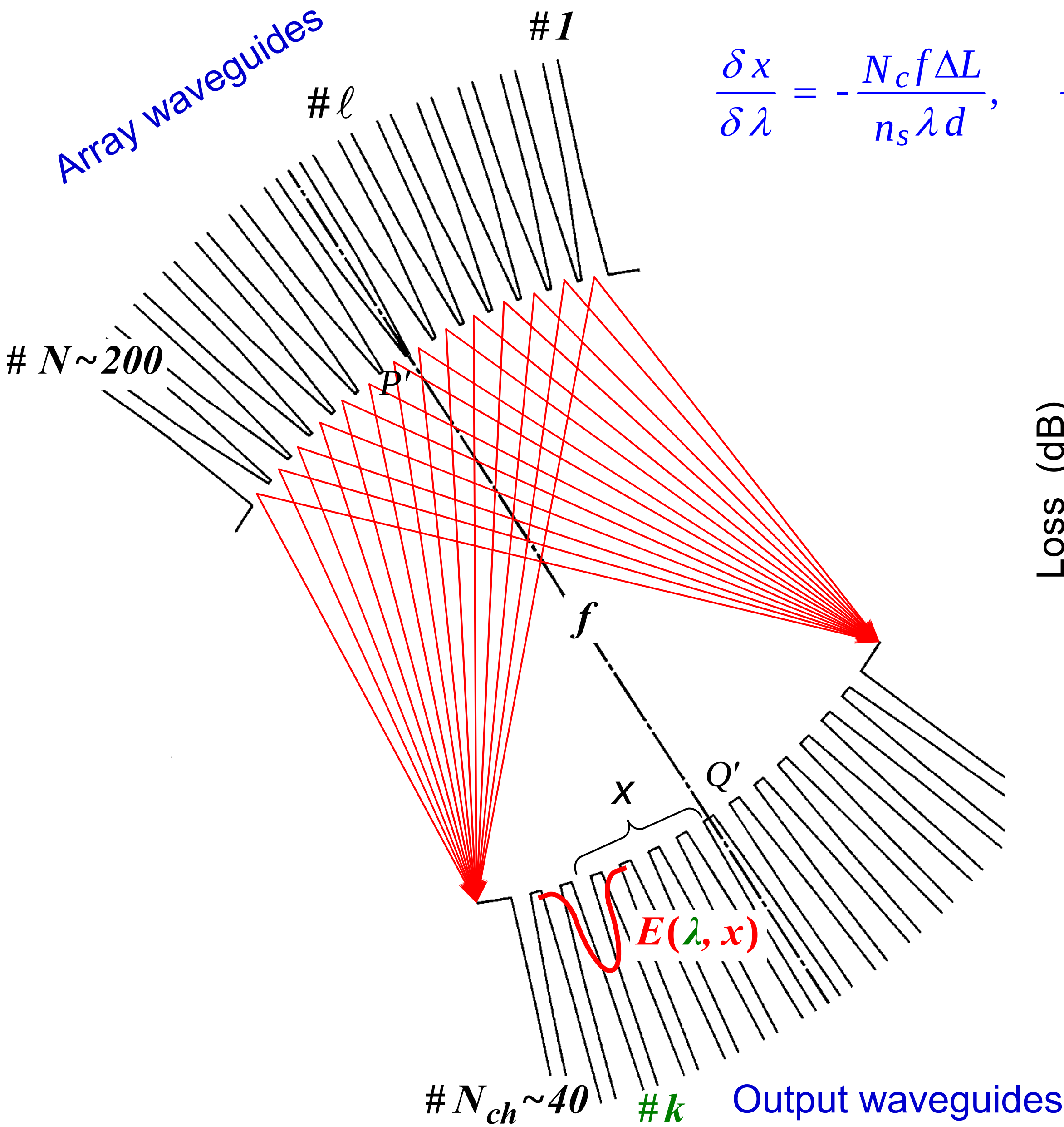


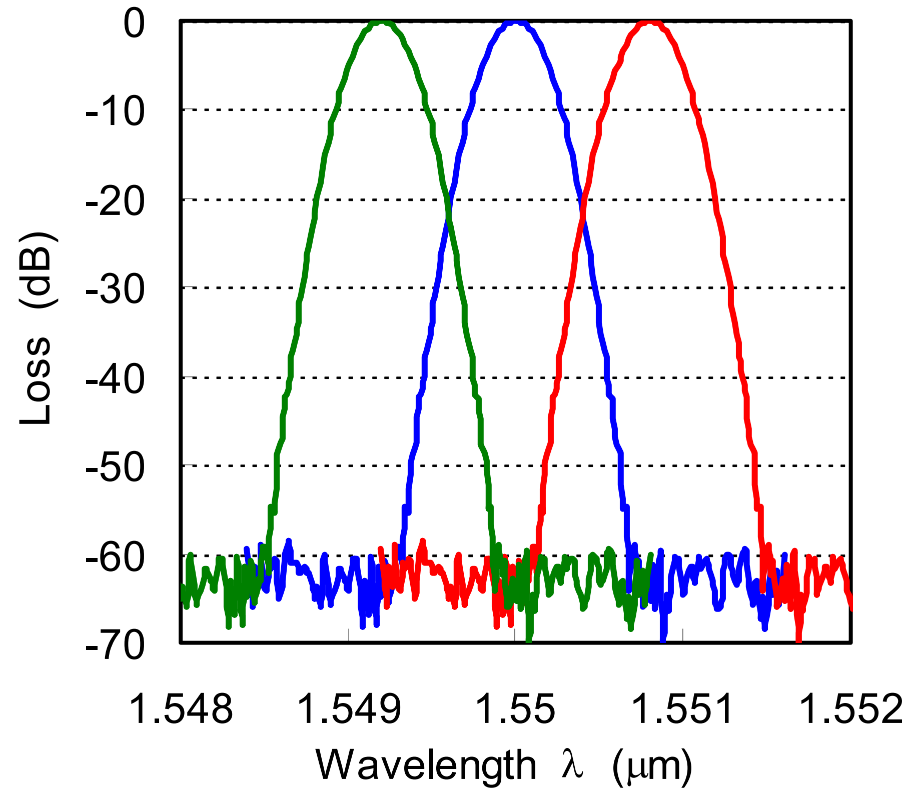
β_s : Propagation constant in Slab

β_c : Propagation constant in Array WG

Multi-beam Interference in the 2nd Slab Region of AWG

$$\frac{\delta x}{\delta \lambda} = -\frac{N_c f \Delta L}{n_s \lambda d}, \qquad \frac{N_c}{n_s} \sim 1.015 \quad \Longrightarrow \quad \frac{\delta x}{\delta \lambda} = -25\,\mu m / 0.8\,nm$$





(a) Focused electric field *E*(*x*) and Local Normal Mode LNM in the output waveguide

Theory of AWG - 1

(1) Interference condition

$$\beta_s(\lambda)\left(\frac{d_1}{f_1}x_1 - \frac{d}{f}x\right) + \beta_c(\lambda)\Delta L = 2m\pi \quad (1)$$

$$\frac{n_c(\lambda)\Delta L}{\lambda} = m \quad (2) \quad \cdots \quad \text{for input/output positions satisfying} \quad \frac{d_1}{f_1}x_1 = \frac{d}{f}x$$

$$n_c(\lambda) = \beta_c(\lambda)/k \quad : \quad \text{Effective-index of arrayWG} \quad (k = 2\pi/\lambda = 2\pi\nu/c)$$

$$n_s(\lambda) = \beta_s(\lambda)/k \quad : \quad \text{Effective-index of slab} \quad (n_s = n_{eff})$$

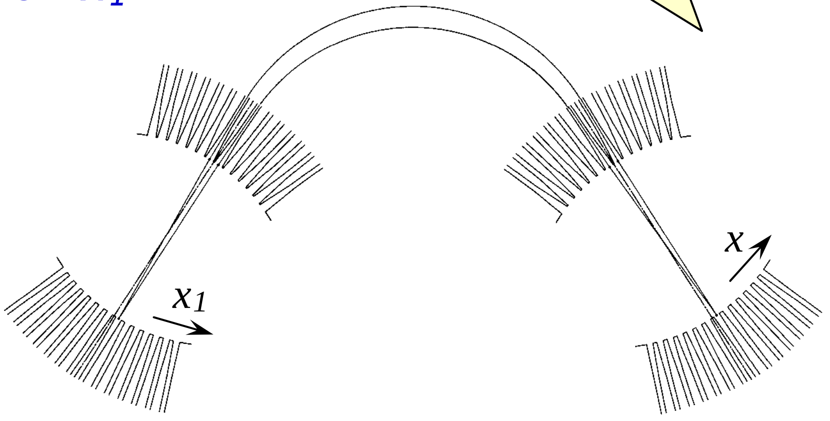
(2) Dispersion of focal point *x* for fixed input position x_1

$$\frac{\delta}{\delta k}\left[\text{left-hand side term of Eq.(1)}\right] = 0$$

$$\frac{\delta\beta_s}{\delta k}\left(\frac{d_1}{f_1}x_1 - \frac{d}{f}x\right) - \beta_s(\lambda)\frac{d}{f}\cdot\frac{\delta x}{\delta k} + \frac{\delta\beta_c}{\delta k}\Delta L = 0$$

II

0



Theory of AWG - 2

$$\frac{\delta x}{\delta k} = -\frac{\lambda}{k}\frac{\delta x}{\delta \lambda} = \frac{\nu}{k}\frac{\delta x}{\delta \nu} = \frac{N_c f \Delta L}{\beta_s d}$$

$$N_c = \frac{\delta \beta_c}{\delta k} = n_c - \lambda \frac{dn_c}{d\lambda} \quad : \text{ Group index}$$

$$\frac{\delta x}{\delta \nu} = \frac{N_c f \, \Delta L}{n_s \nu d} \qquad (3)$$

$$\frac{\delta x}{\delta \lambda} = -\frac{N_c f \Delta L}{n_s \lambda d} \qquad (4)$$

(3) Free spectral range (FSR)

From Eq. (1) for fixed input/output positions x_1 and x

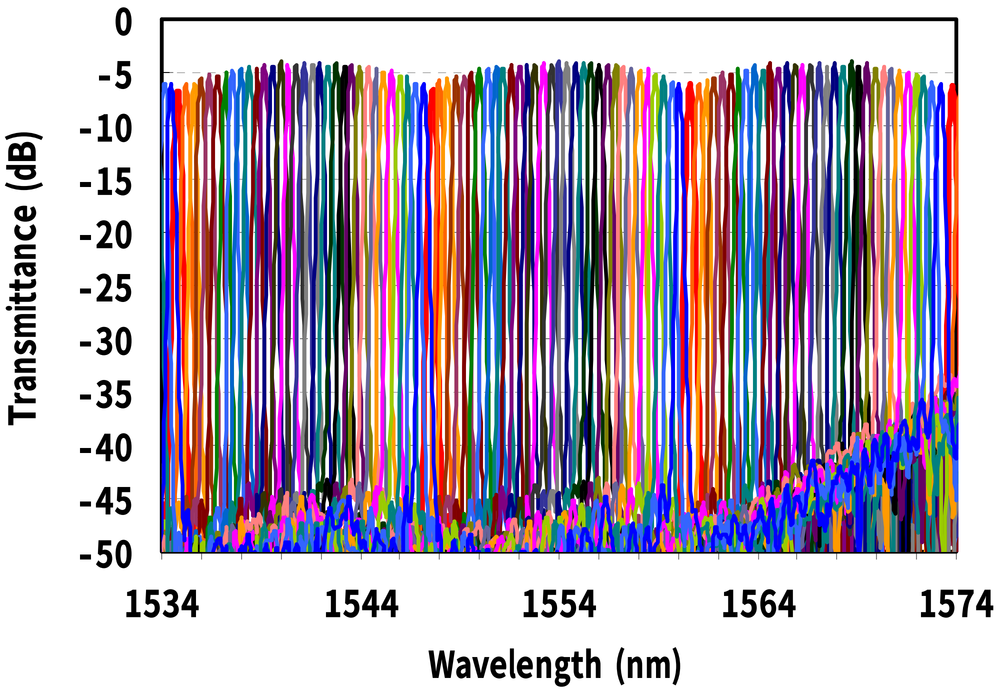
$$\beta_c(\lambda)\Delta L - 2m\pi = \beta_c(\lambda - \lambda_{FSR})\Delta L - 2(m+1)\pi$$

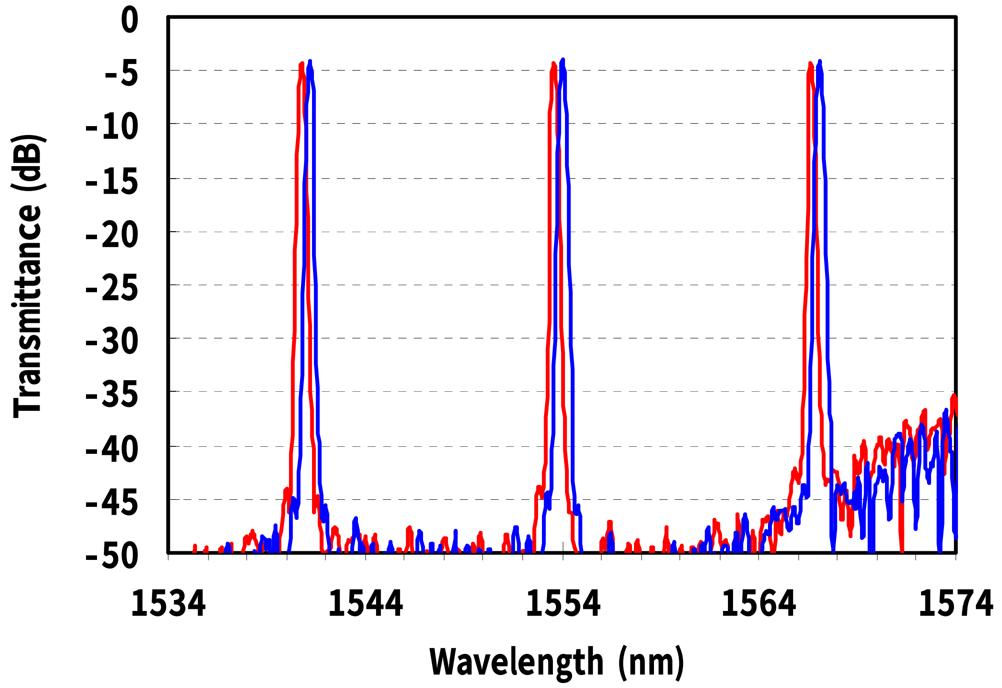
⇩

$$\lambda_{FSR} = \frac{n_c}{N_c} \cdot \frac{\lambda}{m}, \qquad \nu_{FSR} = \frac{n_c}{N_c} \cdot \frac{\nu}{m} \qquad (5)$$

⇩

$$\nu_{FSR} = \frac{c}{N_c \Delta L} \quad \text{[from Eqs.(2) \& (5)]}$$





Theory of AWG - 3

(4) Free spatial range

Spatial separation of the *m*-th and (*m+1*)-th focal positions for the same wavelength λ (fixed x_1)

$$\beta_s(\lambda)\left(\frac{d_1}{f_1}x_1 - \frac{d}{f}x\right) + \beta_c(\lambda)\Delta L - 2m\pi = 0 \qquad (1)$$

$$\beta_s(\lambda)\frac{d}{f}x_m + 2m\pi = \beta_s(\lambda)\frac{d}{f}x_{m+1} + 2(m+1)\pi$$

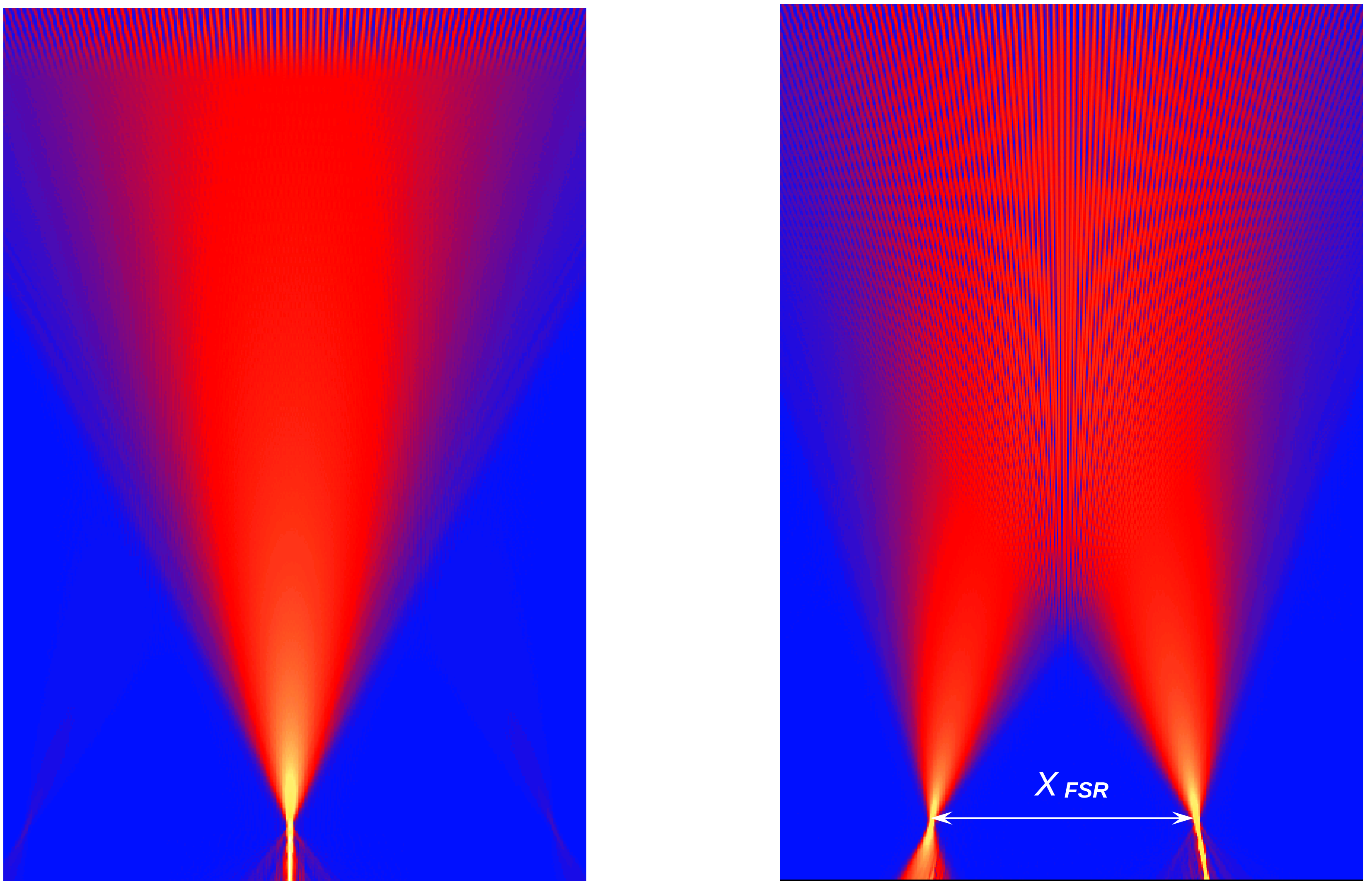
$$\Downarrow$$

$$X_{FSR} = x_m - x_{m+1} = \frac{\lambda f}{n_s d} \qquad (6)$$

(5) Number of available channels

$$N_{ch} = \frac{X_{FSR}}{D} = \frac{\lambda f}{n_s d\, D} \qquad (7)$$

Light Focusing Properties for Center and Off-center Beams



m

(a) $\lambda = \lambda_{center}$

m **+ 1**　　*m*

(b) $\lambda < \lambda_{center}$

Design Procedure of AWG

Given parameters

$$N_{fdm} = 64$$

$$\lambda_0 = 1.55\ (\mu m)$$

$$S_{GHz} = 100\ (GHz)$$

$$D_{stmwg} = 1.5\ (\mu m)$$

$$D_{strcv} = 2.5\ (\mu m)$$

Focal length : Ry

Ry = 518.6 μm
Δ L = 12.1 μm

$$R_y = \frac{n_s D_{stmwg} D_{strcv} N_{fdm}}{\lambda_0}$$

Path length difference Δ*L*

$$\frac{\delta x}{\Delta \nu} = \frac{D_{strcv}}{S_{GHz}} = \frac{N_c R_y \Delta L}{n_s \nu_0 D_{stmwg}}$$

$$\Delta L = \frac{n_s \nu_0 D_{stmwg} D_{strcv}}{N_c S_{GHz} R_y}$$

$$\frac{n_c \Delta L}{\lambda_0} = m$$

$$\hat{m} = \left| \frac{n_c \Delta L}{\lambda_0} \right| \quad \Rightarrow \quad \Delta \hat{L} = \hat{m} \frac{\lambda_0}{n_c} \quad \Rightarrow \quad \hat{R}_y = \frac{n_s D_{stmwg} D_{strcv} \nu_0}{N_c S_{GHz} \Delta \hat{L}}$$

Farfield Pattern given by Fresnel-Kirchhoff Diffraction Formula

$$G(\rho) = \int_{-\infty}^{\infty} E(x) \exp\left(j \frac{2\pi n_s}{\lambda} x \tan\rho \right) dx$$

$$\theta = \frac{n_s \tan\rho}{\lambda} \sim \frac{n_s}{\lambda}\rho$$

$$G(\theta) = \int_{-\infty}^{\infty} E(x) \exp\left(j\, 2\pi\theta x \right) dx$$

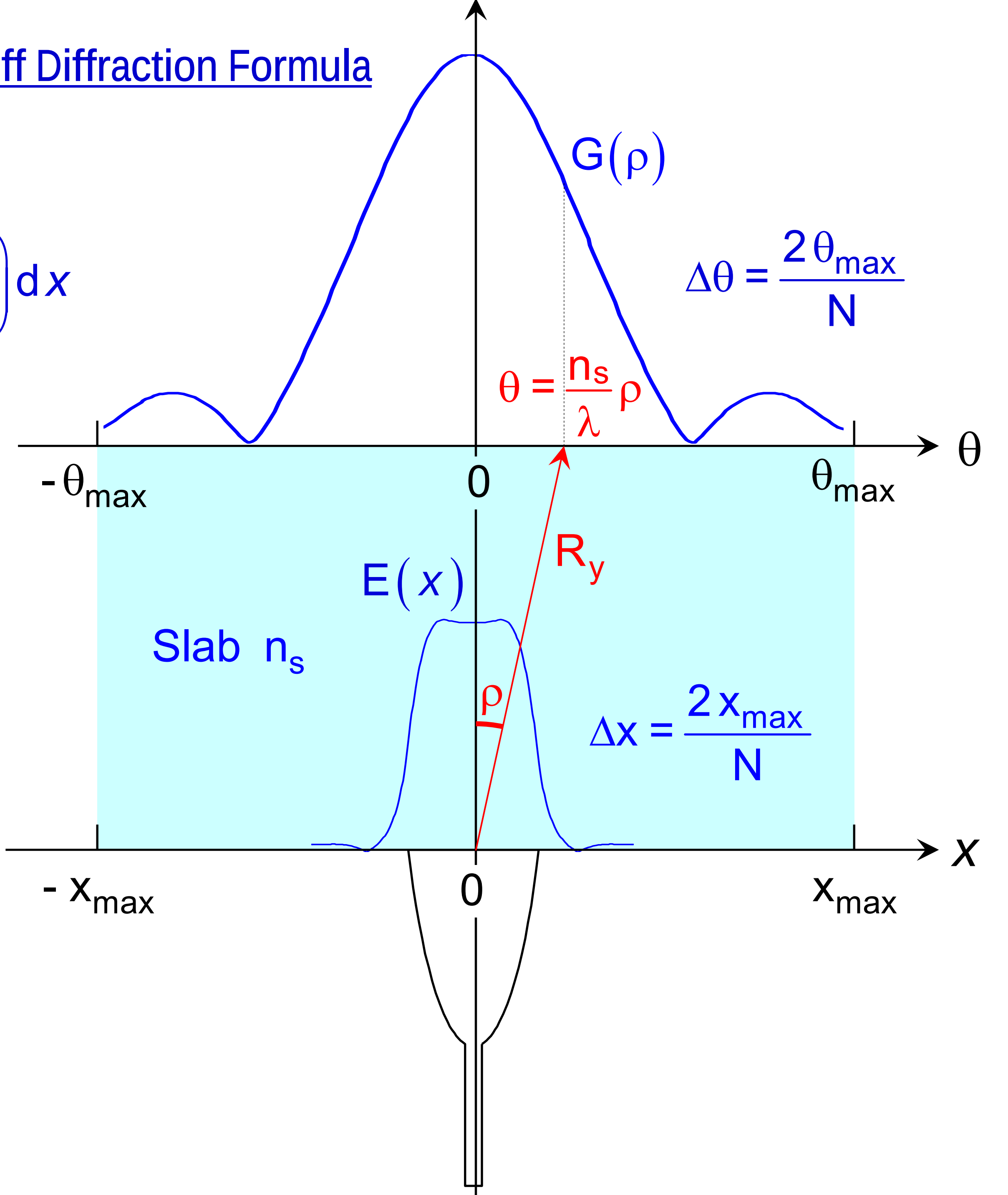
Discretization

$$G_n = G(n\Delta\theta)$$

$$E_\ell = E(\ell\Delta x)$$

$$\Delta x = \frac{1}{2\theta_{max}}$$

$$\Delta\theta = \frac{1}{2x_{max}}$$



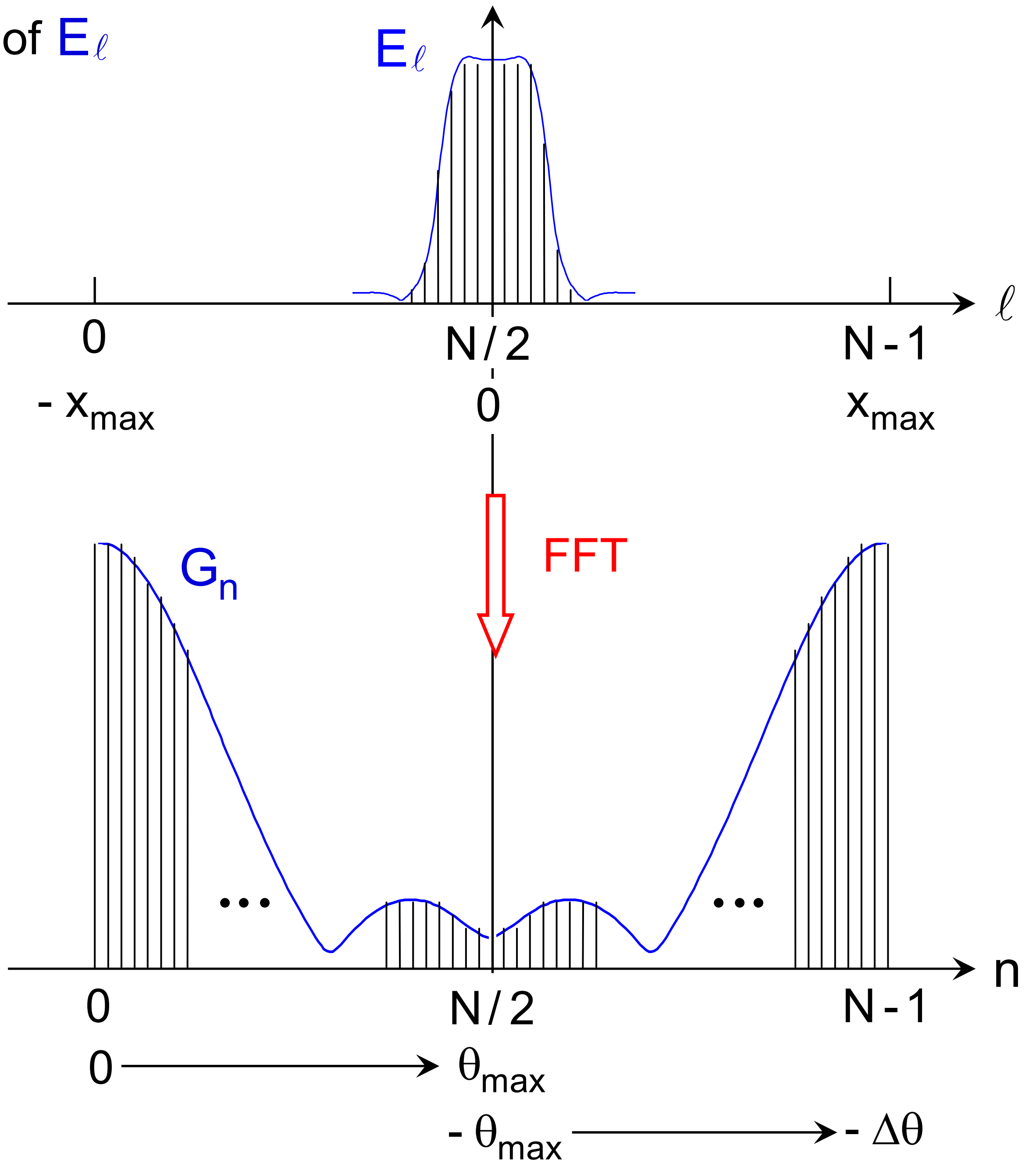
Farfield Pattern is given by the FFT of E_ℓ

$$G_n = \frac{1}{\sqrt{N}} \sum_{\ell=0}^{N-1} E_\ell \, e^{j 2\pi \frac{n\ell}{N}}$$

$$\Delta\rho = \frac{\lambda}{n_s} \Delta\theta \quad \left(\text{from } \theta = \frac{n_s}{\lambda} \rho \right)$$

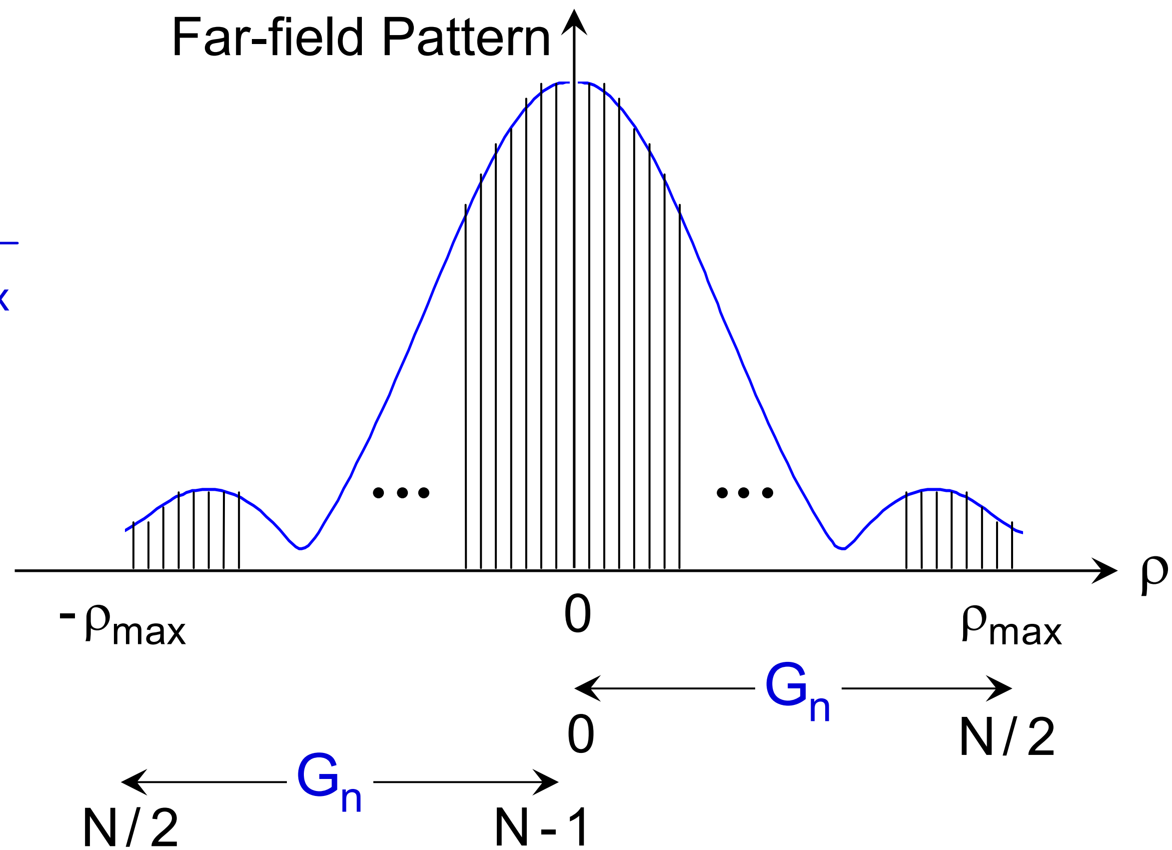
$$\rho_{max} = \frac{N}{2} \Delta\rho \qquad \theta_{max} = \frac{N}{2} \Delta\theta$$

$$\rho_{max} = \frac{\lambda}{n_s} \theta_{max}$$



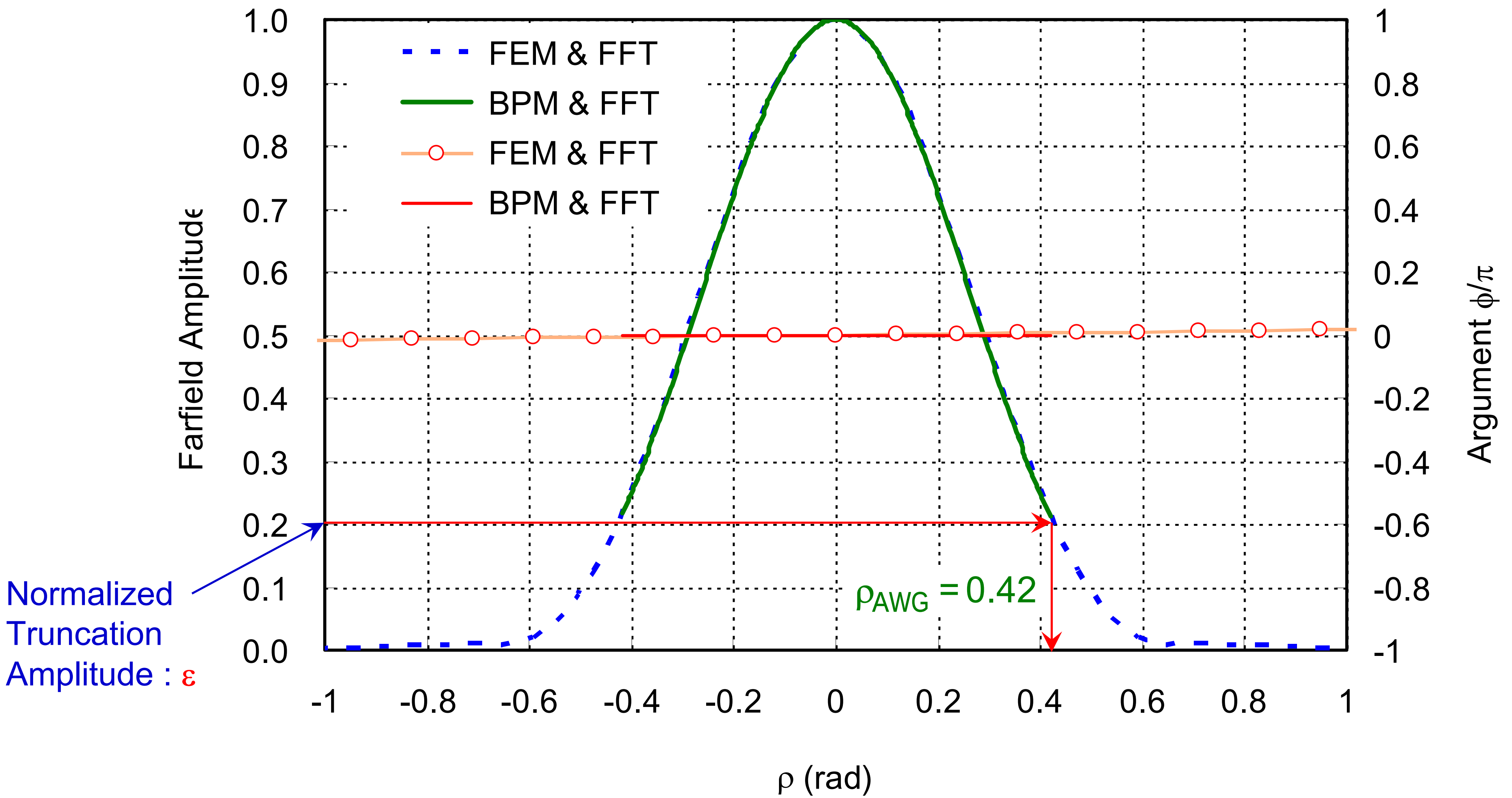
$$\Delta\rho = \frac{\lambda}{n_s}\Delta\theta = \frac{\lambda}{n_s}\frac{1}{2x_{max}}$$

$$\rho_{max} = \frac{N}{2}\Delta\rho$$

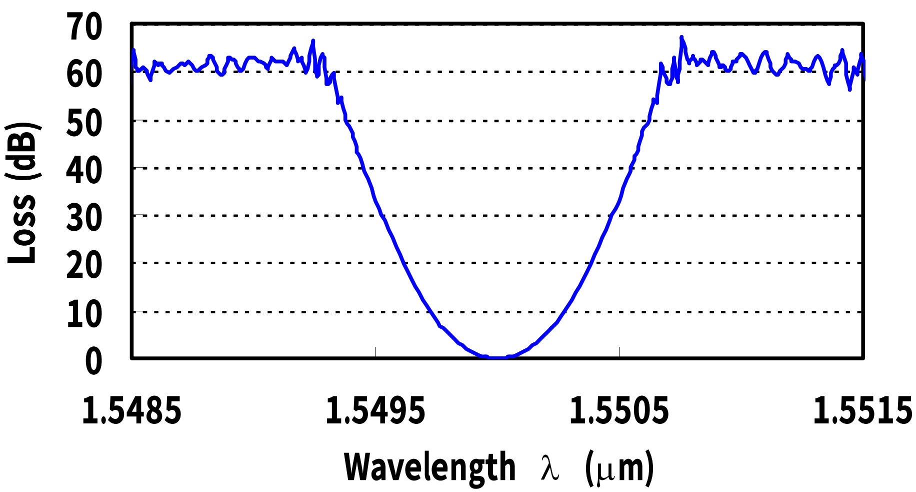


Far-field Pattern from the Input Waveguide

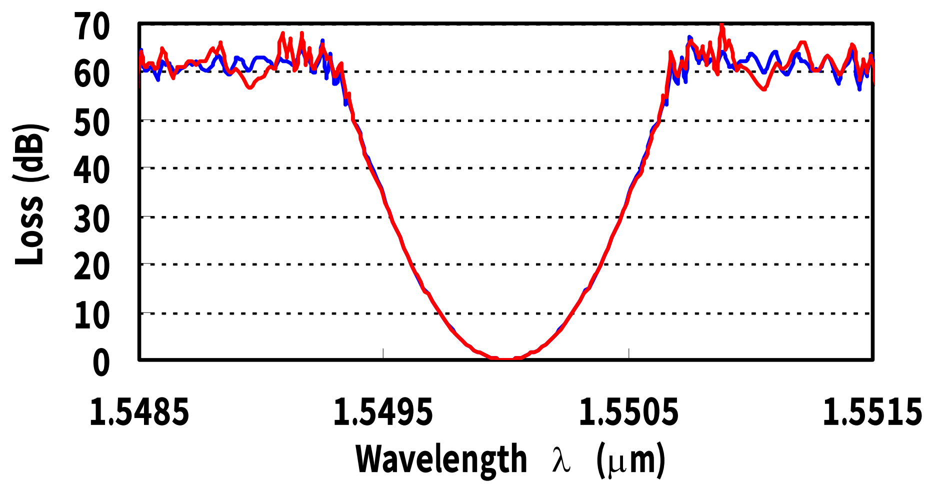
$$E(x) = \exp\left(-\frac{2x^2}{w_{x1}^2} - \frac{2x^4}{w_{x2}^4}\right) \rightarrow FFT\left[E(x)\right] \quad (N = 1024)$$



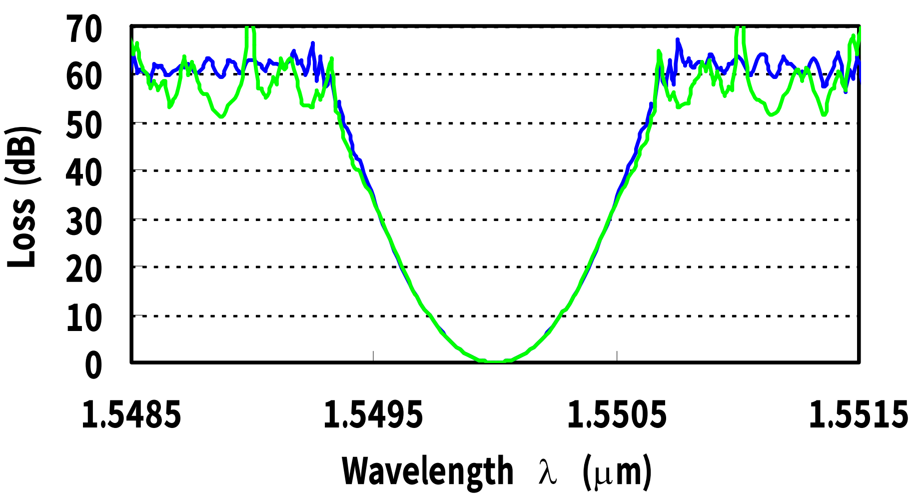
Theoretical Crosstalk vs NTA ε : (BPM Calculation)



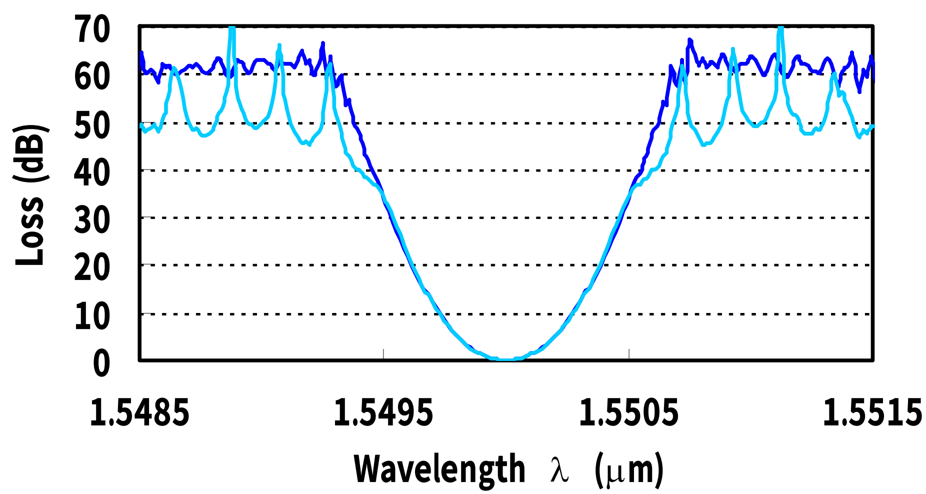
(a) NTA ε = 0.0085



(b) NTA ε = 0.08



(c) NTA ε = 0.134



(d) NTA ε = 0.19

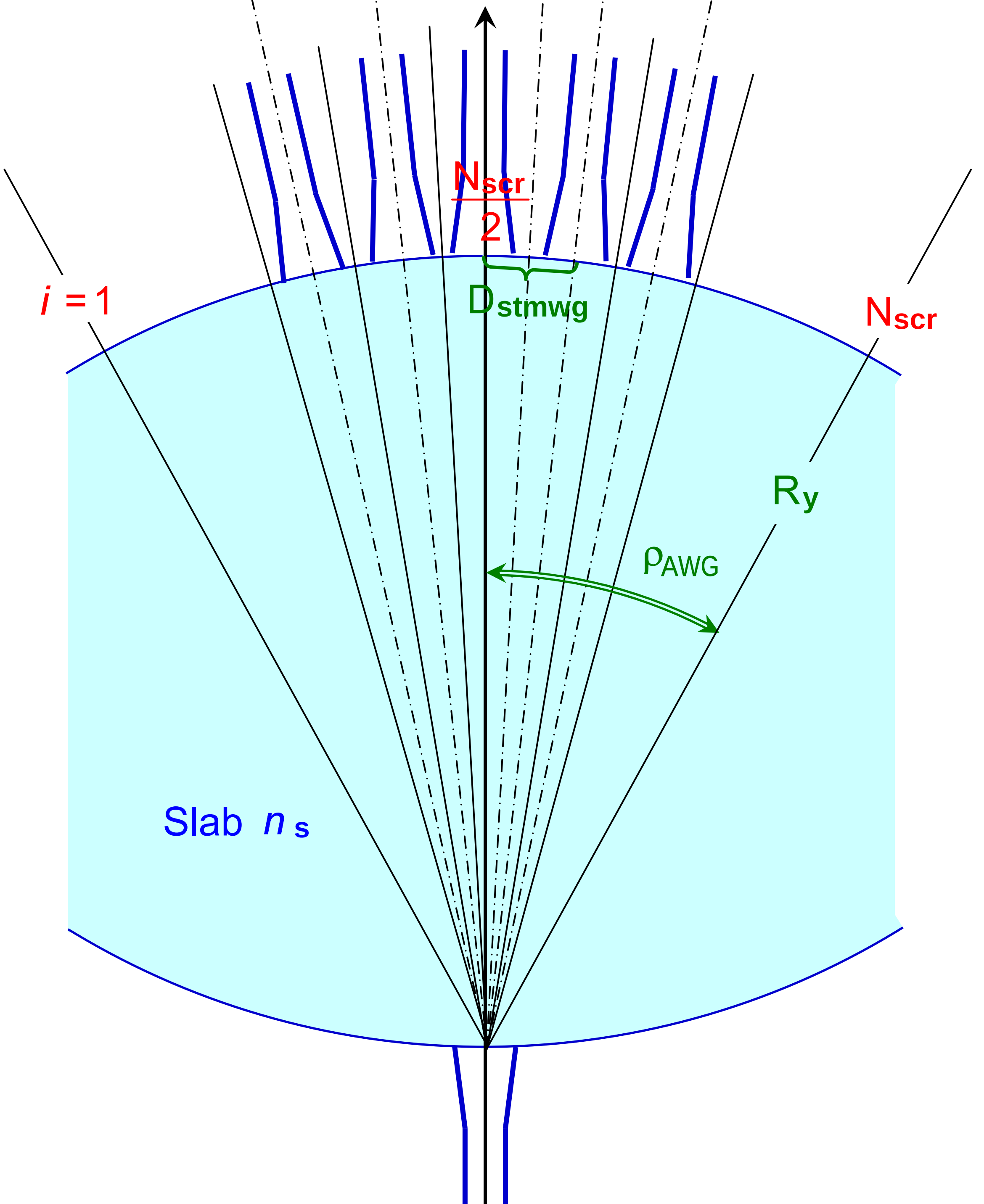
$$\rho_{AWG} = 0.42$$

⇓

$$R_y \cdot 2\rho_{AWG} = N_{scr} \cdot D_{stmwg}$$

⇓

$$N_{scr} \sim \frac{R_y \cdot 2\rho_{AWG}}{D_{stmwg}}$$

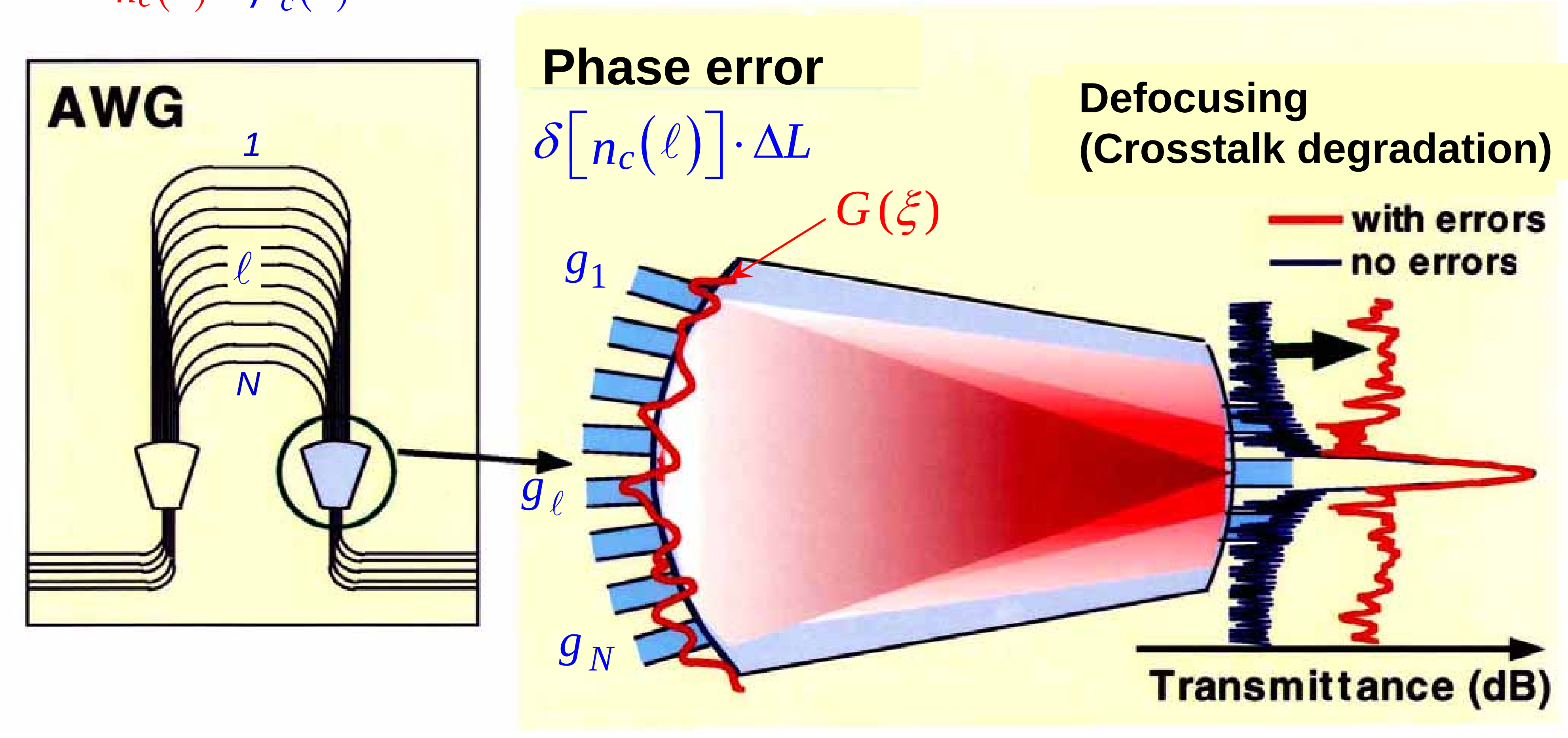


Crosstalk of AWG Caused by Fabrication Error

$$g_\ell = a_\ell \cdot \exp\left[-jk\,n_c(N-\ell)\Delta L\right]$$

$$n_c(\ell) = \beta_c(\ell)/k$$

$$\sum_{\ell=1}^{N} F\left[g_\ell\right] = F\left[\sum_{\ell=1}^{N} g_\ell\right] = F\left[G(\xi)\right]$$



Light focusing in 2nd slab region

Waveguide Layout for AWG Design

Parameters to be determined

α : Angle of slab $\angle PQQ'$

L_{slab} : Distance between Q-Q'

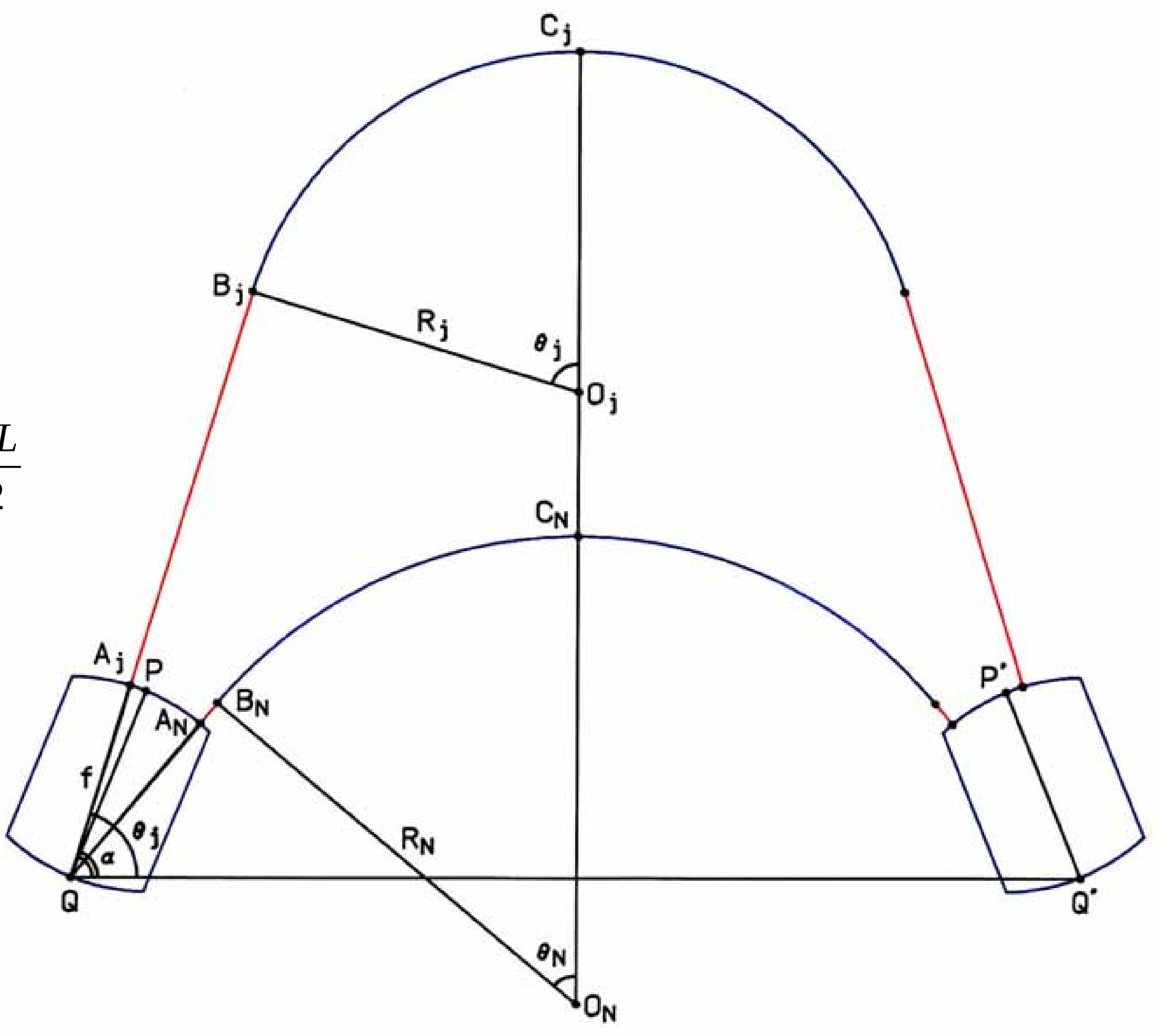
Basic equations

$$S_j + R_j \cdot \theta_j = S_N + R_N \cdot \theta_N + (N - j)\frac{\Delta L}{2}$$

$$\left(f + Y_{fnl} + S_j\right)\cos\theta_j + R_j \sin\theta_j = \frac{L_{slab}}{2}$$

s_j : Distance between A_j - B_j

s_N : Predetermined distance between A_N – B_N



Equations to Determine AWG Configuration

(2N-1) equations for (2N+1) unknown values

$\alpha, \quad L_{slab}$

$$R_N = \frac{\frac{L_{slab}}{2} - \left(f + Y_{fnl} + S_N\right)\cos\theta_N}{\sin\theta_N} \qquad (8)$$

$$R_j = \frac{\frac{L_{slab}}{2} - \left[f + Y_{fnl} + S_N + R_N \cdot \theta_N + (N-j)\frac{\Delta L}{2}\right]\cos\theta_j}{\sin\theta_j - \theta_j \cdot \cos\theta_j} \quad (j = 1 \sim N-1) \qquad (9)$$

$$S_j = S_N + \left(R_N \cdot \theta_N - R_j \cdot \theta_j\right) + (N-j)\frac{\Delta L}{2} \quad (j = 1 \sim N-1) \qquad (10)$$

Choose solution under the following conditions

$Min\left(R_j\right) \geq R_{\min}$ (given by bending loss)

$S_j \geq 0$ (taper length)

Waveguide separation $\geq s_{\min}$ (given by mode coupling)

Layout of Array Waveguides

$$\hat{\phi}_0 = \sin^{-1}\left(\frac{D_{stmwg}/2}{\hat{R}_y}\right)$$

$$\hat{\psi}_j = \left(N_{scr}+1-2j\right)\cdot\hat{\phi}_0$$

$$\hat{\sigma}_0 = \sin^{-1}\left[\frac{\hat{D}_{strcv}\cdot\left(1+\frac{V_{nrGHz}}{S_{GHz}}\right)/2}{\hat{R}_y}\right]$$

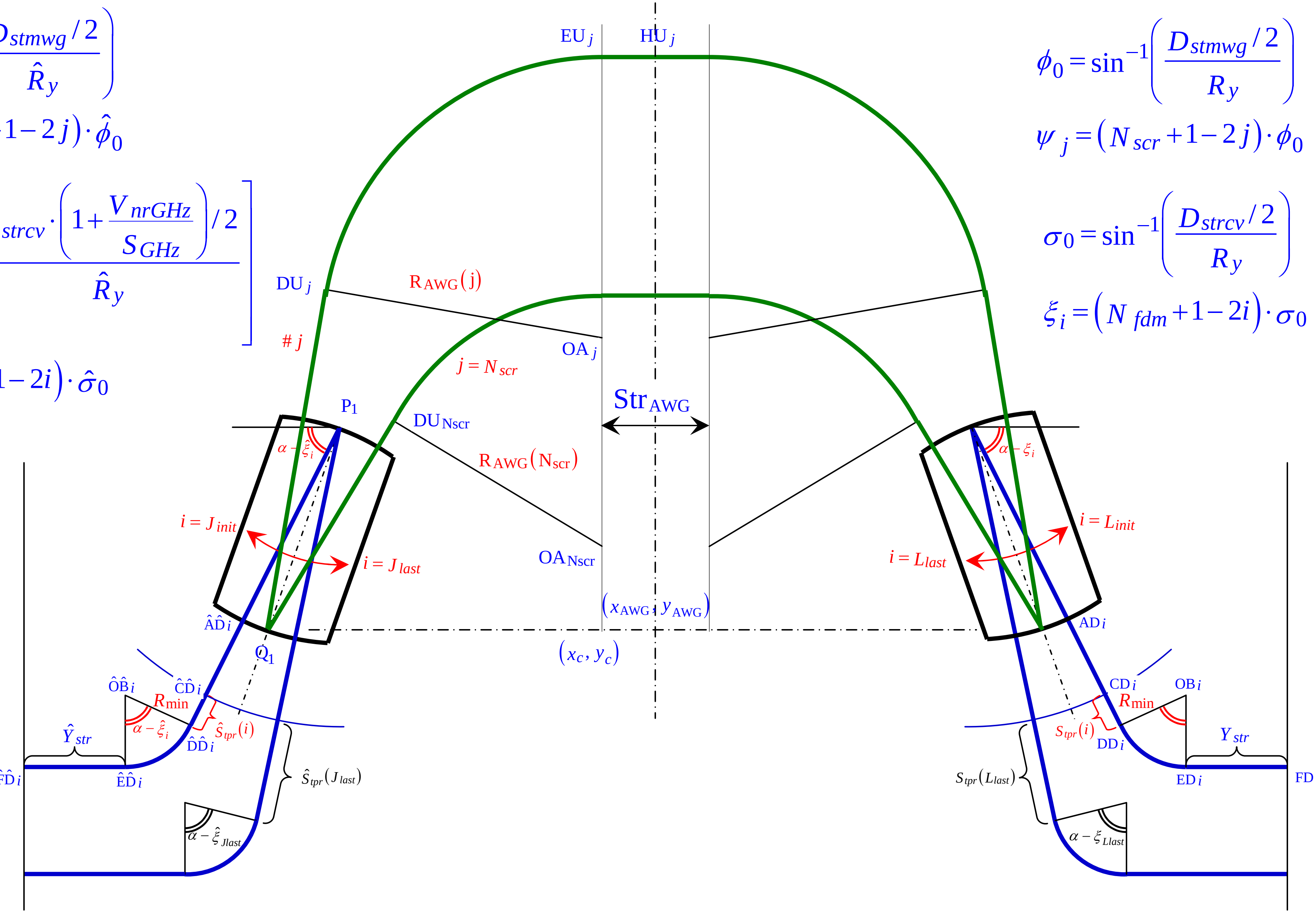
$$\hat{\xi}_i = \left(N_{fdm}+1-2i\right)\cdot\hat{\sigma}_0$$

$$\phi_0 = \sin^{-1}\left(\frac{D_{stmwg}/2}{R_y}\right)$$

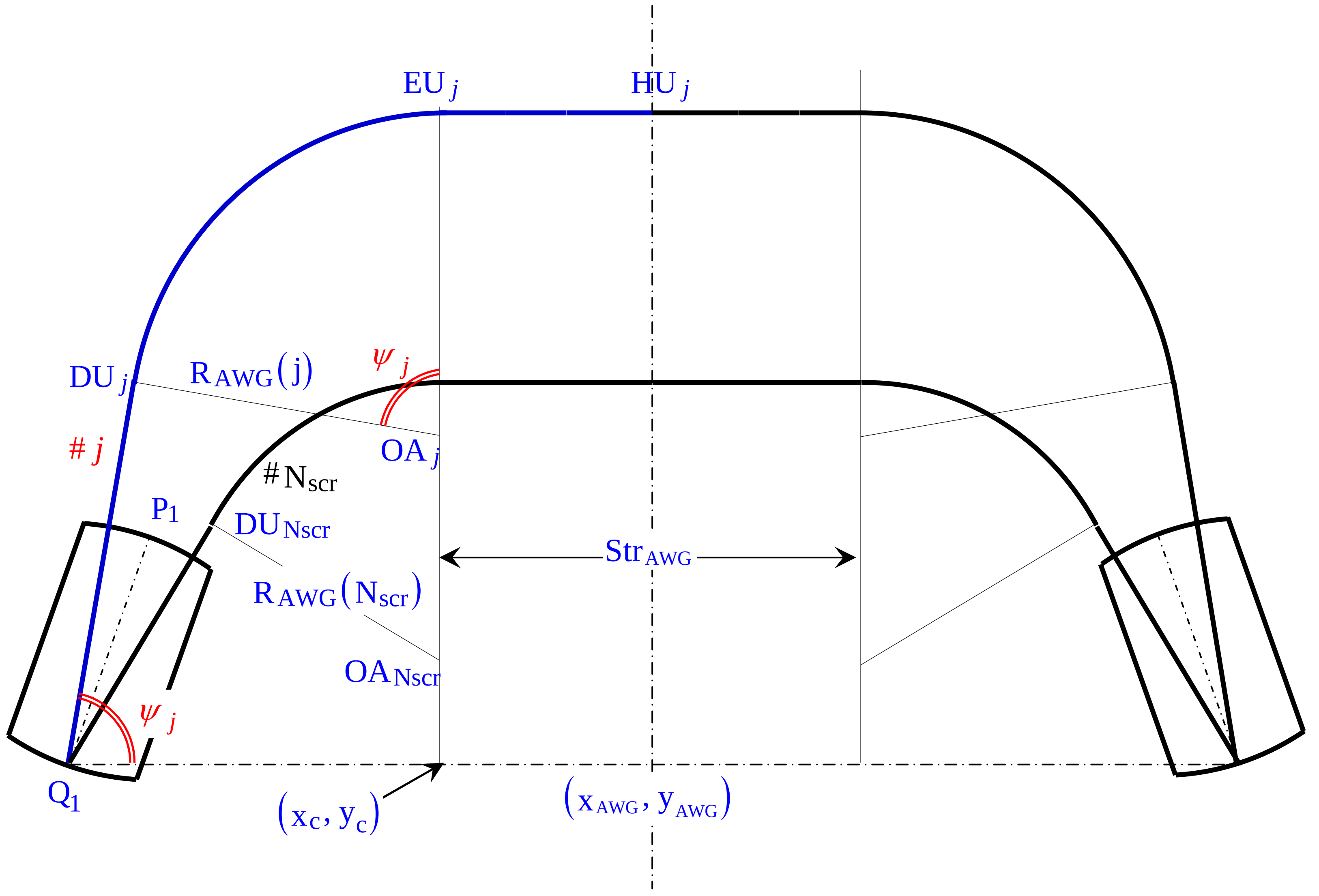
$$\psi_j = \left(N_{scr}+1-2j\right)\cdot\phi_0$$

$$\sigma_0 = \sin^{-1}\left(\frac{D_{strcv}/2}{R_y}\right)$$

$$\xi_i = \left(N_{fdm}+1-2i\right)\cdot\sigma_0$$



Layout of Array Waveguides

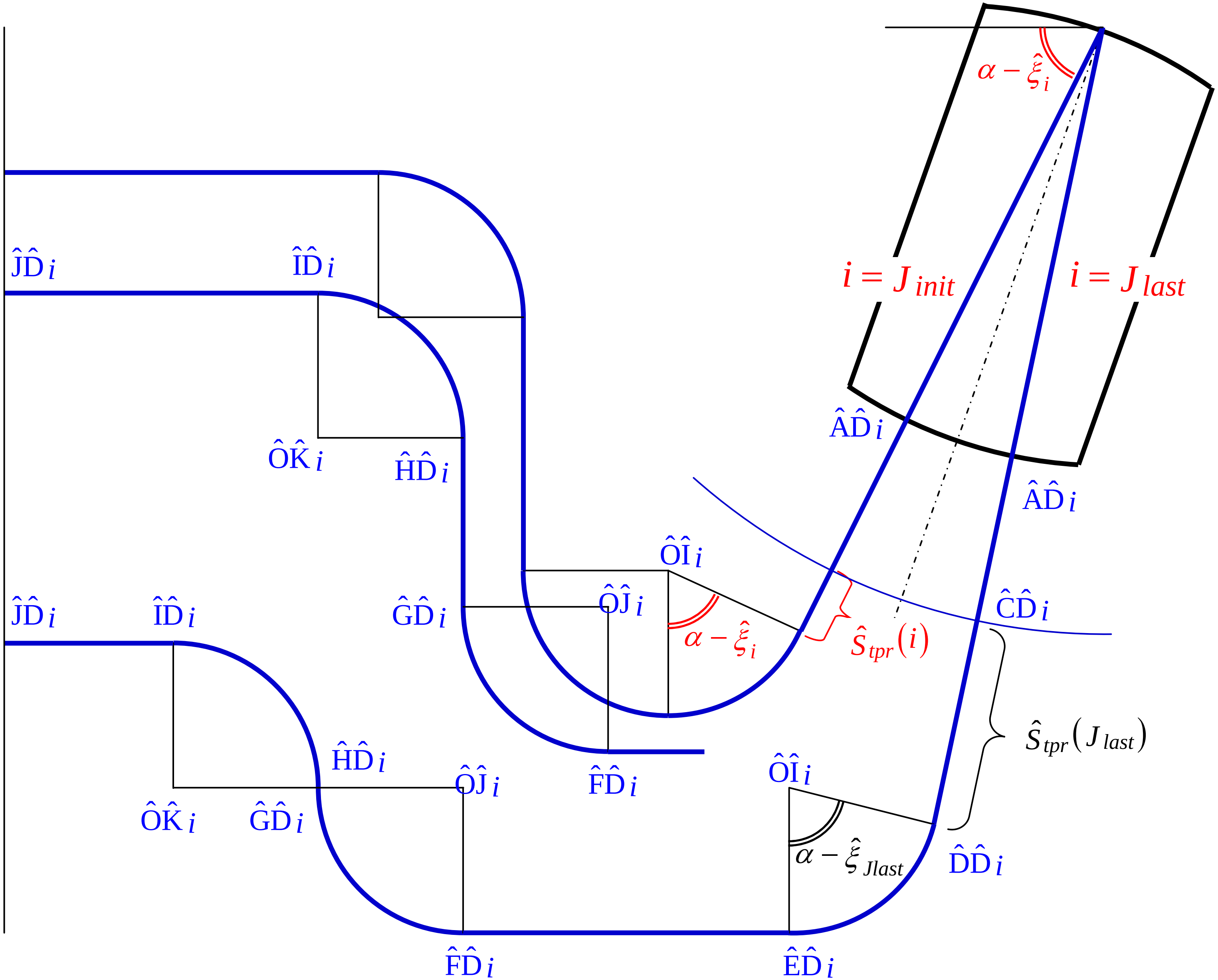


Layout of AWG : Vernier Input WGs

$$\hat{D}_{strcv} = D_{strcv}\left(1+\frac{V_{nrGHz}}{S_{GHz}}\right)$$

$$\hat{\sigma}_0 = \sin^{-1}\left(\frac{\hat{D}_{strcv}}{2R_y}\right)$$

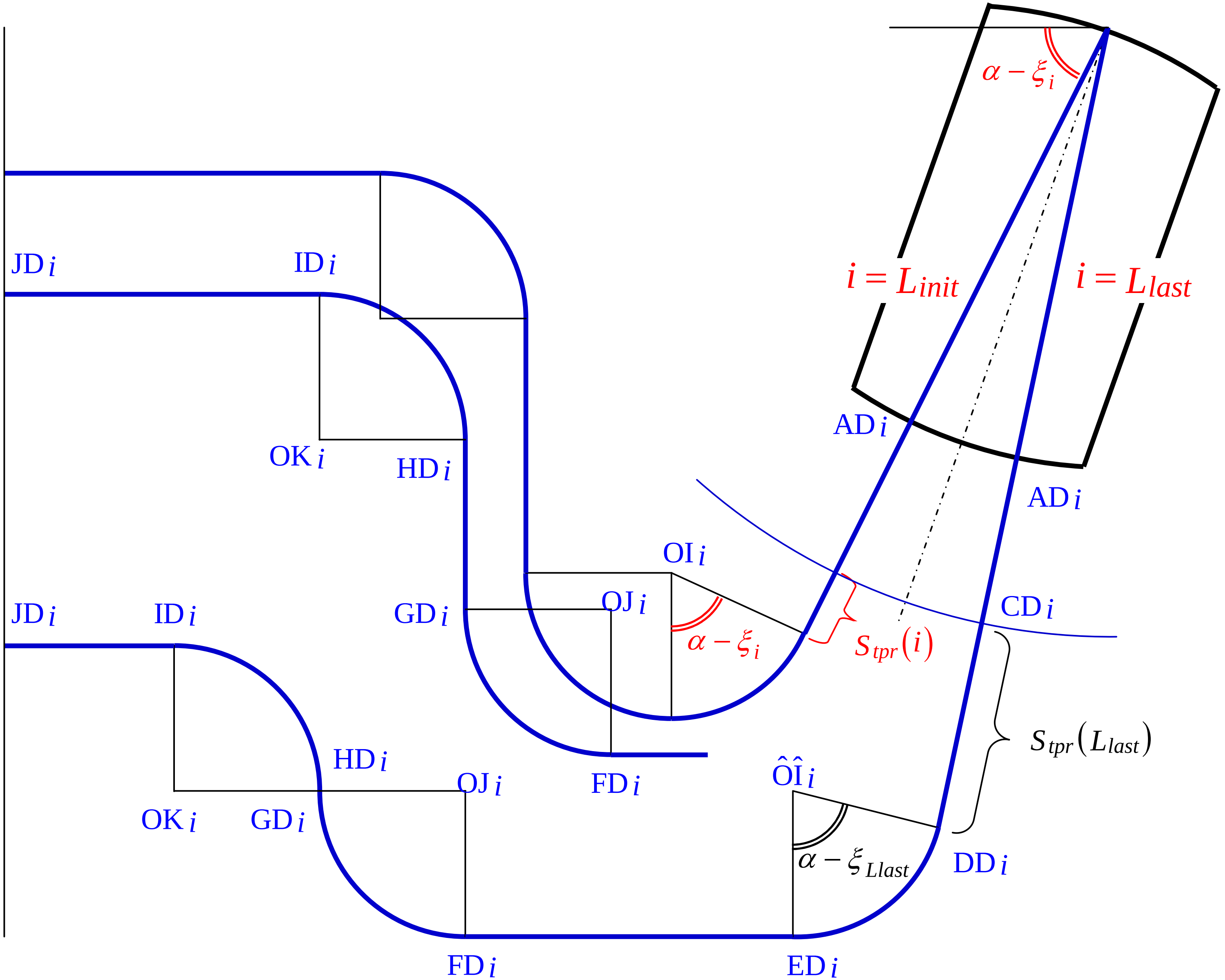
$$\hat{\xi}_i = \left(N_{fdm}+1-2i\right)\cdot\hat{\sigma}_0$$



Layout of AWG : Normal Output WGs

$$\sigma_0 = \sin^{-1}\left(\frac{D_{strcv}}{2R_y}\right)$$

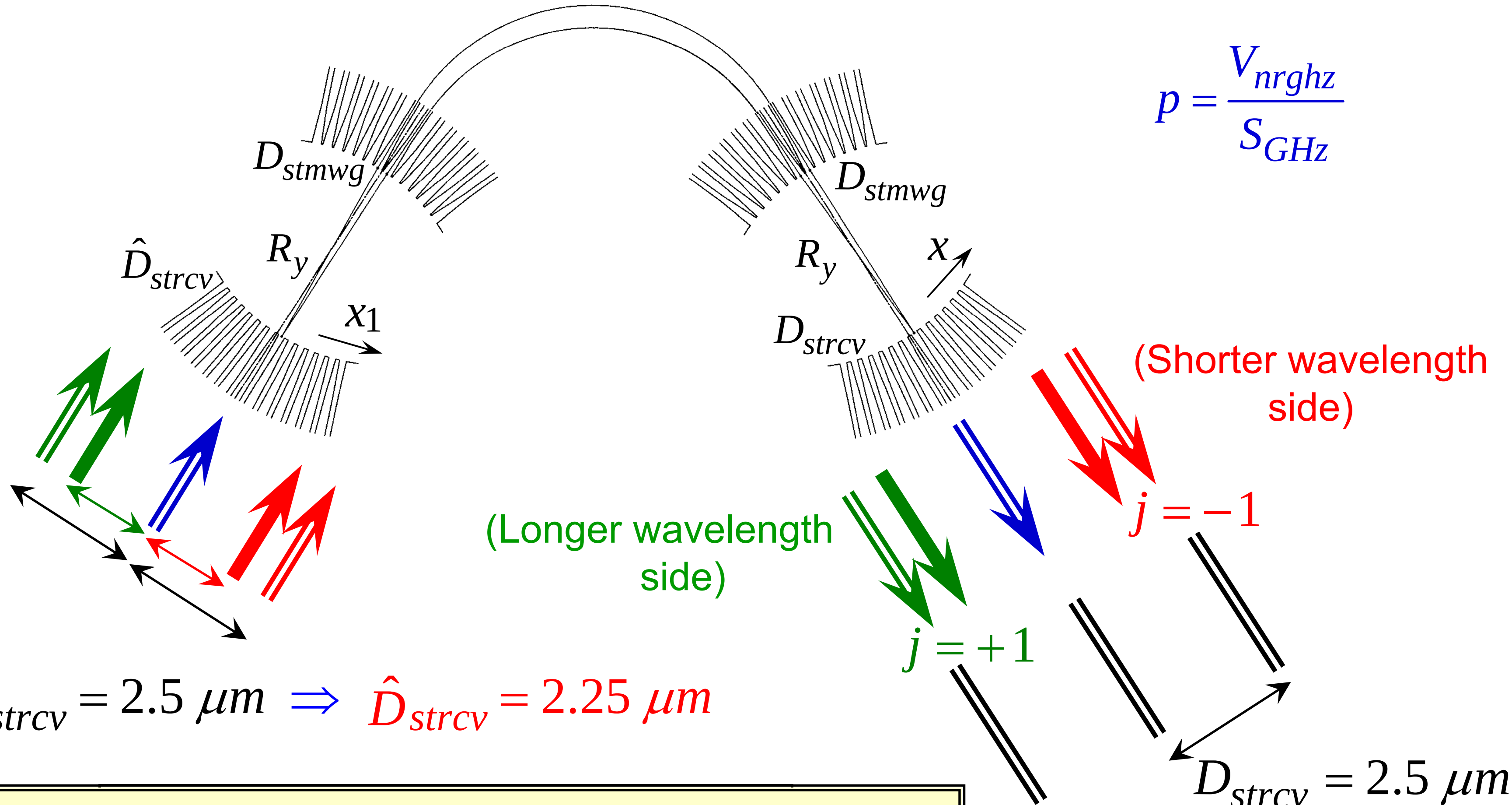
$$\xi_i = \left(N_{fdm} + 1 - 2i\right)\cdot\sigma_0$$



Principle of Vernier Center Wavelength Trimming

$$\hat{D}_{strcv} = D_{strcv}\left(1 - \frac{V_{nrGHz}}{S_{GHz}}\right) \Rightarrow \hat{D}_{strcv} = D_{strcv}\left(1 + \frac{V_{nrGHz}}{S_{GHz}}\right)$$ in the current mask

$$p = \frac{V_{nrghz}}{S_{GHz}}$$



$$\lambda(j) = \lambda_{center} + j\,p\,S_{GHz} = \lambda_{center} + jV_{nrghz}$$

30-days Trial version

Absoft Pro Fortran : http://www.absoft.com/evalform.html

AutoCAD LT :
http://usa.autodesk.com/adsk/servlet/oc/offer/form?siteID=123112&id=9247811

MathType : http://www.dessci.com/en/products/mathtype/trial.asp

How to use Pro Fortran

(1) Start Absoft Compiler → Absoft Developer Tools Interface → “No” → New Project → “Cancel”

(2) Configure → Set Default Options → “New” tab

F77 →

General → quiet

Compatibility → Fold to Upper Case

Miscellaneous → Promote REAL and COMPLEX

Format → Wide Format

Plug-ins → IMSL Library

“OK” → Save as “double-r8” → “double-r8.gui” stored in [Absoft Work Folder] → “Close”

(Note I) Setting should be -q –N113 –W –N109

(Note II) An Instruction of IMSL Library is in [Documentation] ... MATH.pdf etc

How to use Pro Fortran

(3) Store “double-r8.gui” in → [Absoft70] → [BIN] → [Resources] → [IDE Resources] → [Settings]

(4) Prepare File with the Name “f.f”

Double Click “double-r8.gui” → “No” → Configure → Set Options

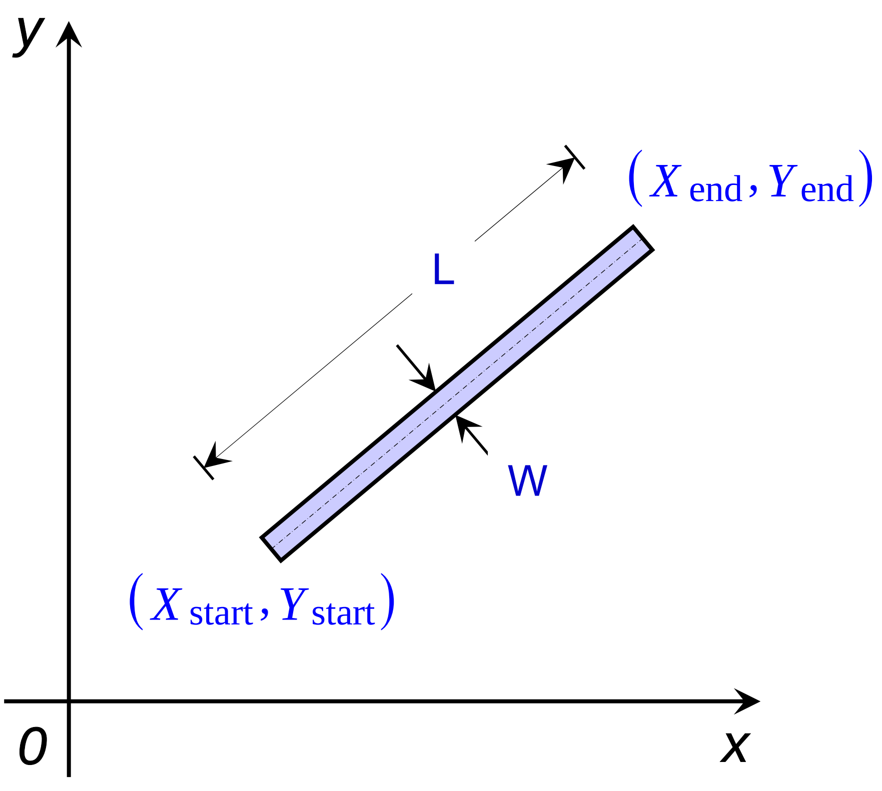
Target Directory → Click “…” → C: → [Absoft70] → Click “Absoft Work Folder” → “OK”

“Ctrl + F” → Double Click “f.f” → “Close” → Click “+” → “Ctrl + B” → “f.exe”

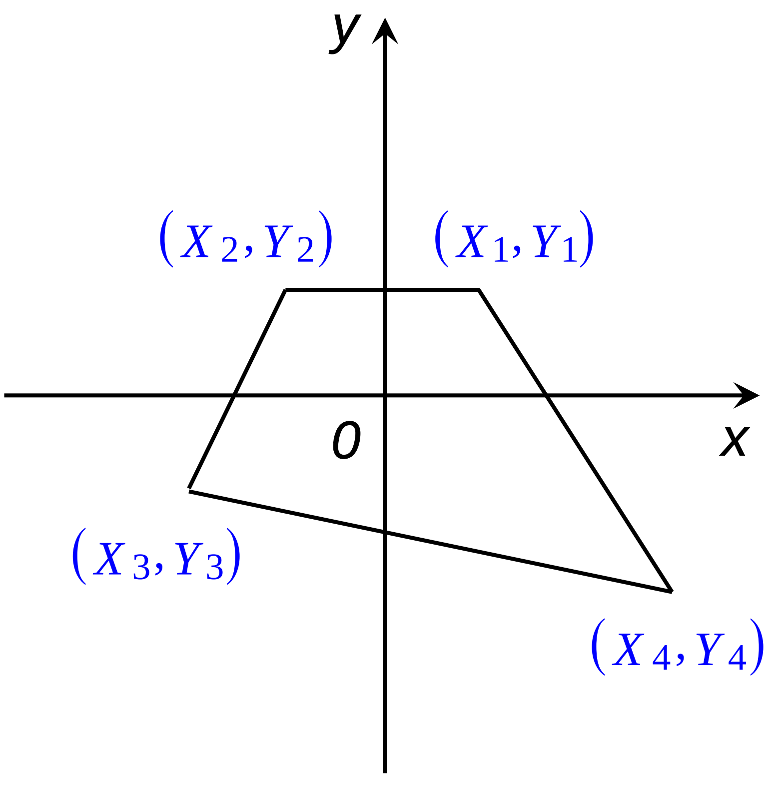
(5) “Ctrl + E” → Program Executed → Remove “f.f” → “Close” → Save changes to “double-r8.gui” ?

→ “Yes”

Mask Data Generation Procedure

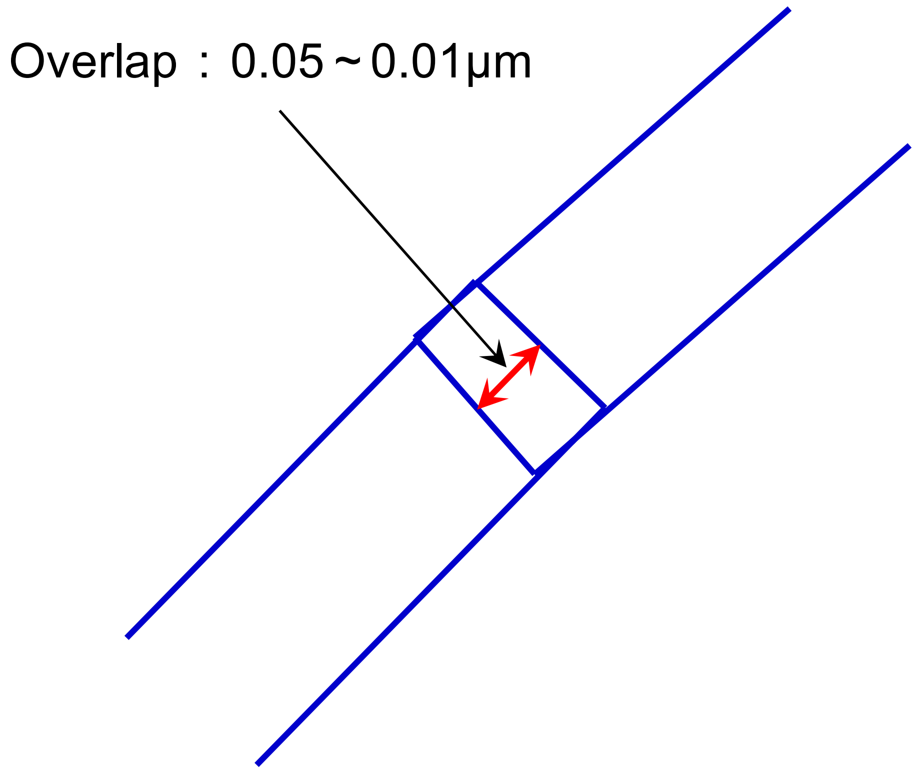


(a) Generation of “Line”



(b) Generation of “Quadrangle”

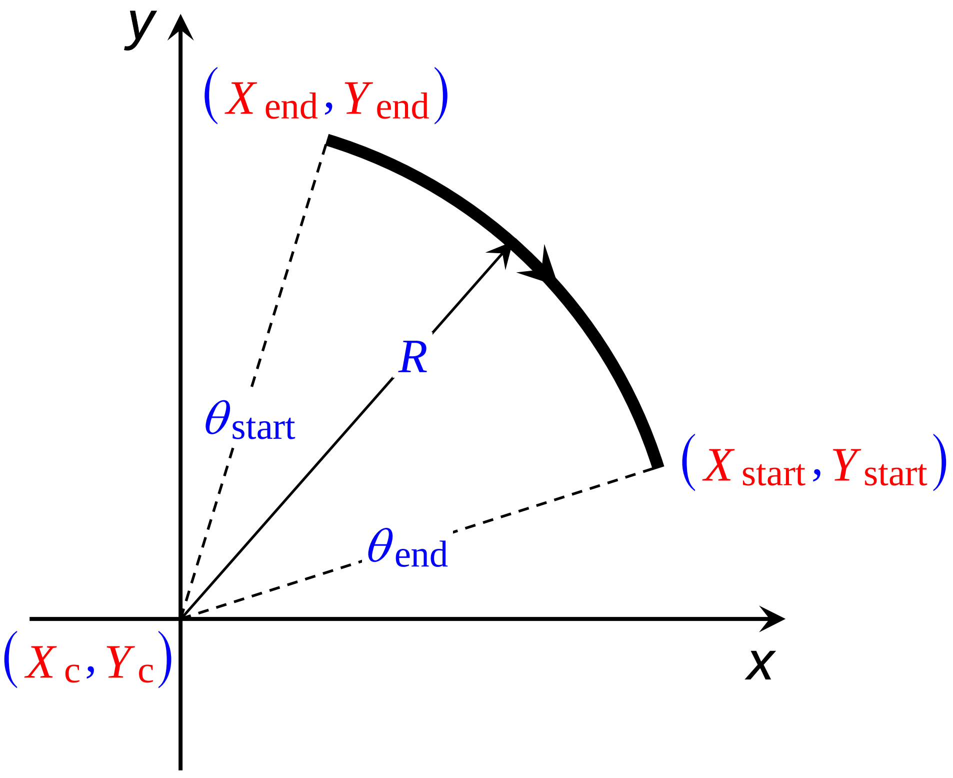
Mask Data Generation Procedure



(a) Overlap of Line's

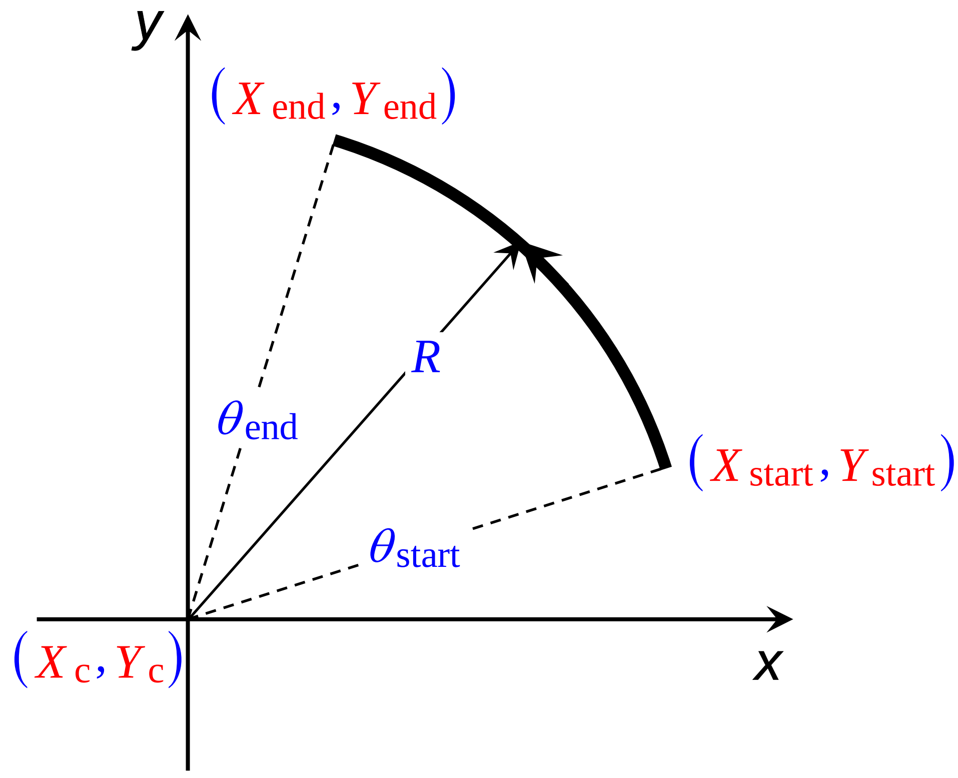
DXF File Generation Procedure

$$\tan\left(\frac{\theta_{\mathrm{start}}-\theta_{\mathrm{end}}}{4}\right)$$



(a) Arc in clockwise direction

$$\tan\left(\frac{\theta_{\mathrm{end}}-\theta_{\mathrm{start}}}{4}\right)$$



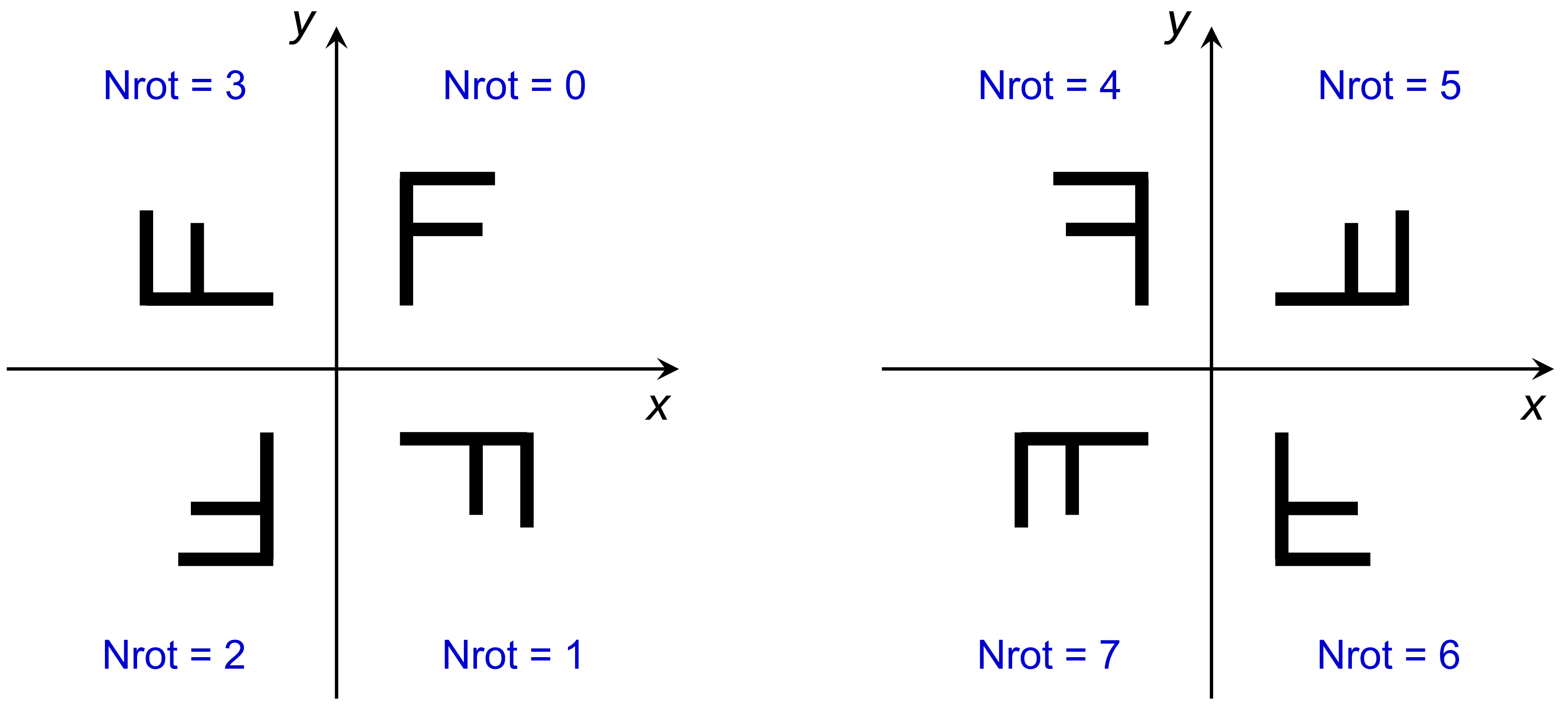
(b) Arc in counterclockwise direction

Mask Company → Mask Minimum Division Angle

Minimum Division Angle

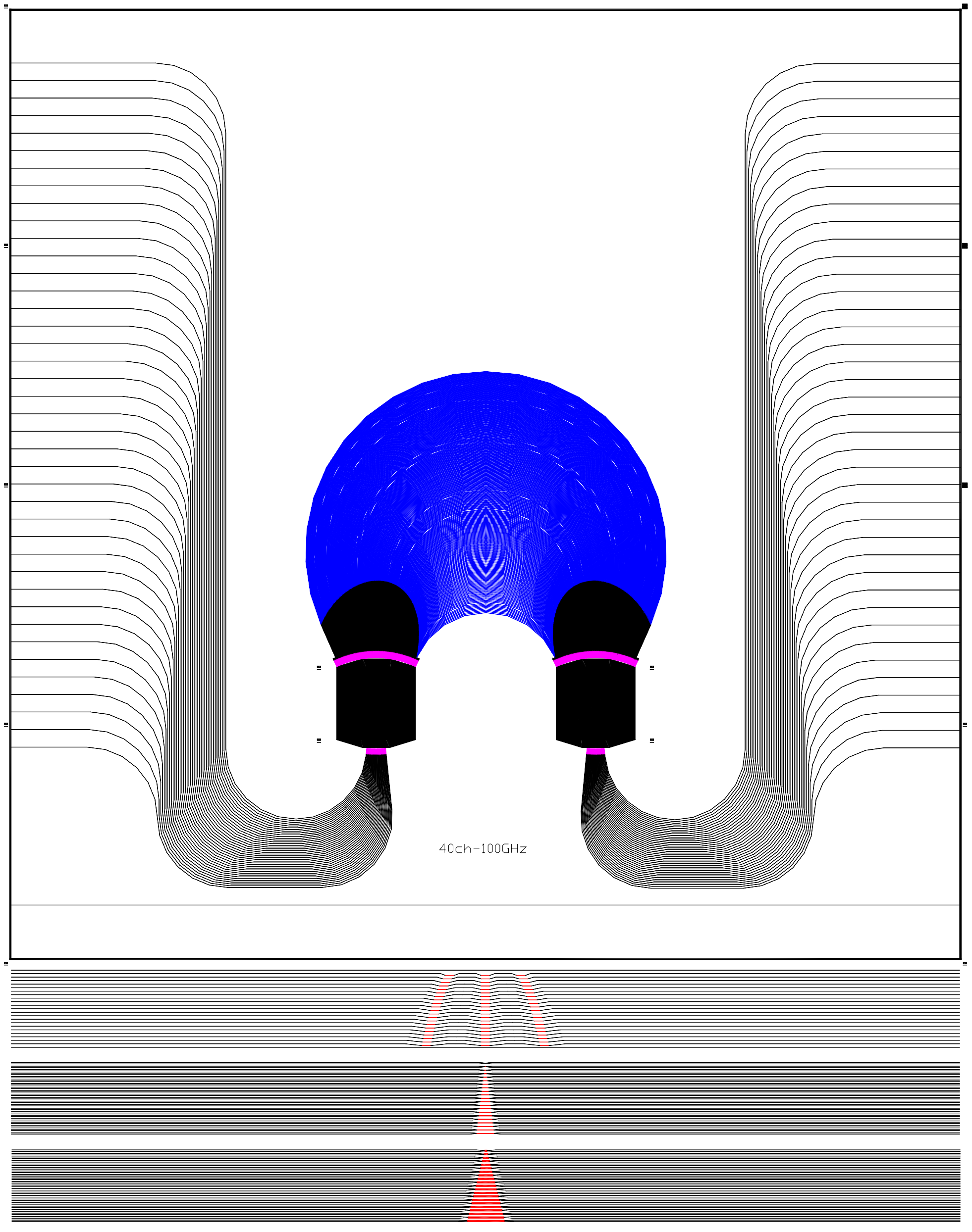
WG Type	R (μm)	$\Delta\theta$ (Deg)	$\Delta\alpha$ (rad)	R*$\Delta\alpha$ (μm)
HΔ-PLC	2000 ~ 5000	0.1	0.0017	3.4 ~ 8.5
InP & Si-Rib	50 ~ 500	0.1	0.0017	0.085 ~ 0.85
Si-Wire	5	0.1	0.0017	0.0085

Rotation of the Cell



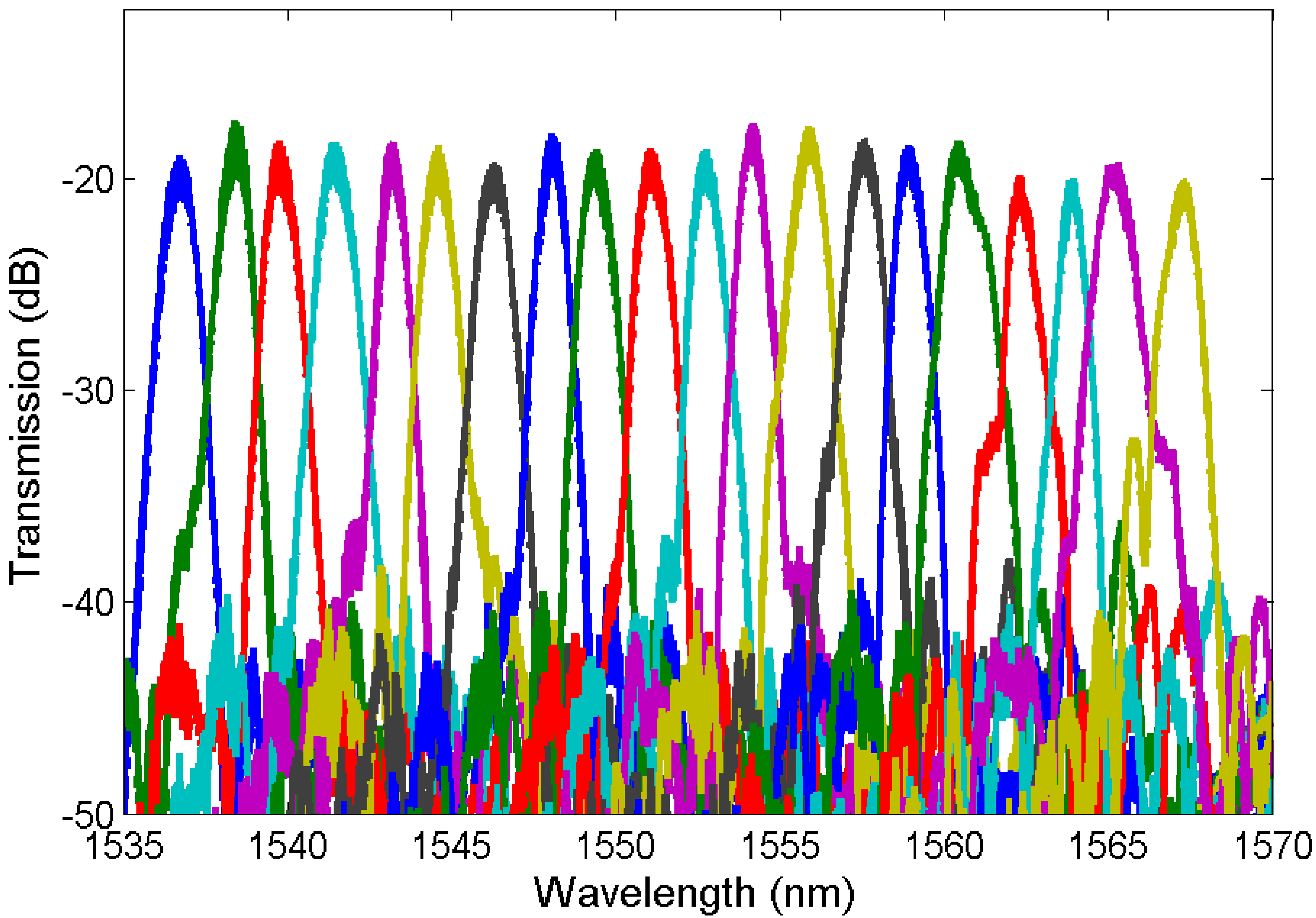
Chip Layout of AWG : 40ch-100GHz

X_{dev} = 5.4 mm
Y_{dev} = 5.4 mm



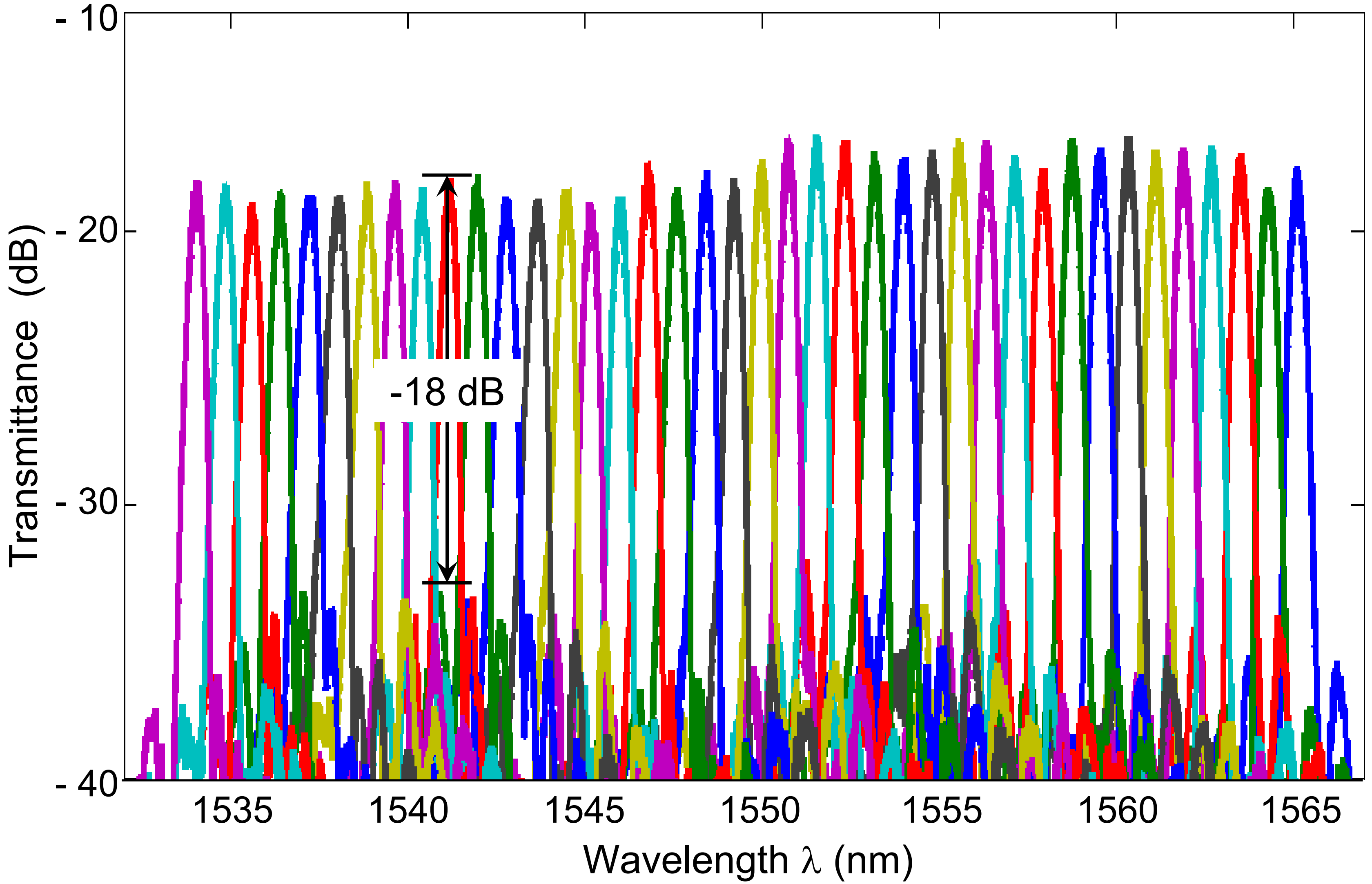
Demux Properties of Si-Rib AWG : 20ch-200GHz

Crosstalk ~ -20 dB



Demux Properties of Si-Rib AWG : 40ch-100GHz

N = 290, ΔL = 12.1 μm, L_{av} = 1.43 mm   Crosstalk ~ -18 dB



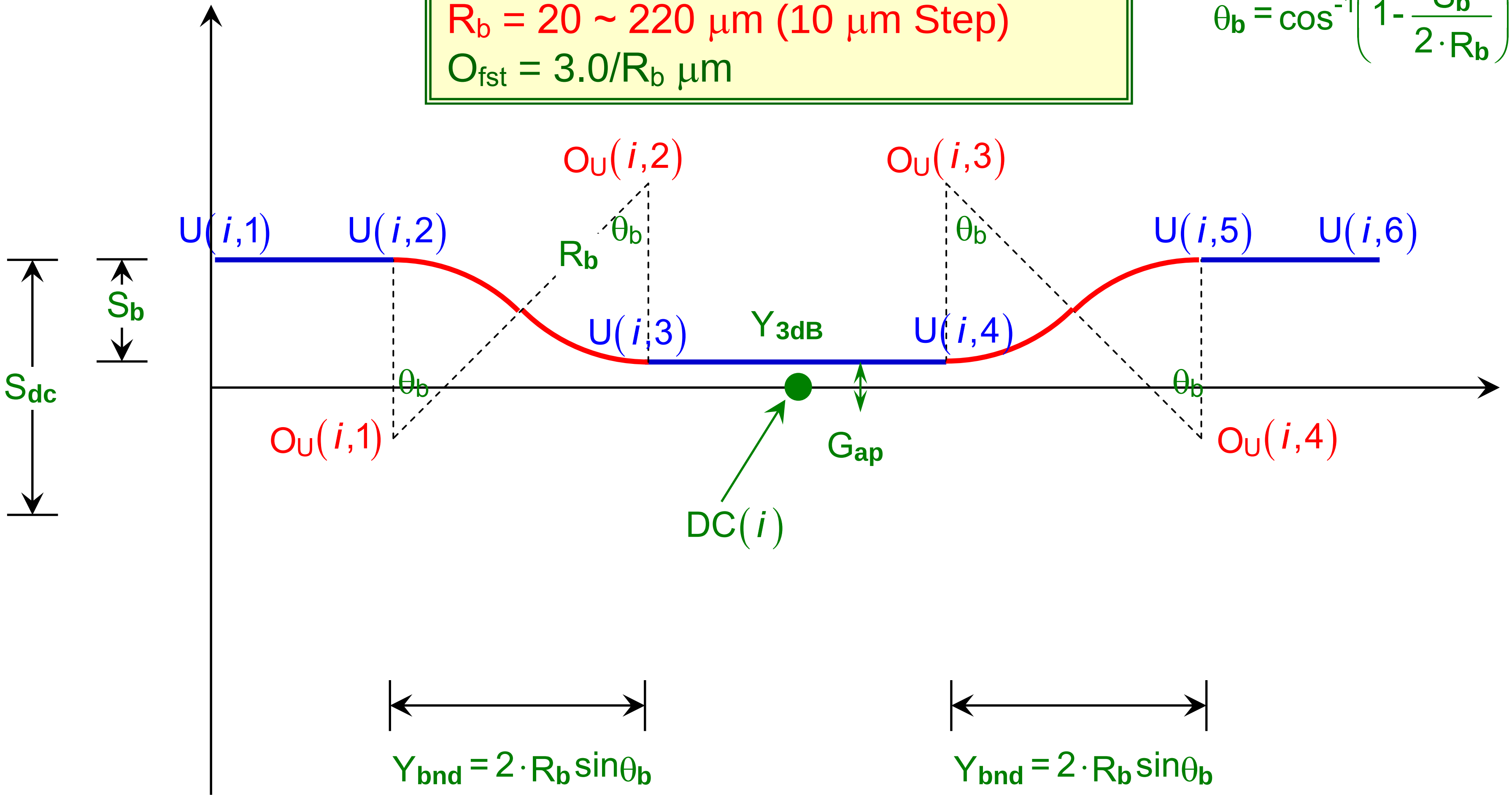
Test Waveguides

Test Radius of Curvature by S-Bends

S_{dc} = 21.2 μm
G_{ap} = 0.2 μm (Core edge separation)
S_b = 10.0 μm
Y_{3dB} = 10 μm
R_b = 20 ~ 220 μm (10 μm Step)
O_{fst} = 3.0/R_b μm

$$S_b = \frac{S_{dc}}{2} - \left(\frac{G_{ap}}{2} + a\right)$$

$$\theta_b = \cos^{-1}\left(1 - \frac{S_b}{2 \cdot R_b}\right)$$



$$Y_{bnd} = 2 \cdot R_b \sin\theta_b$$

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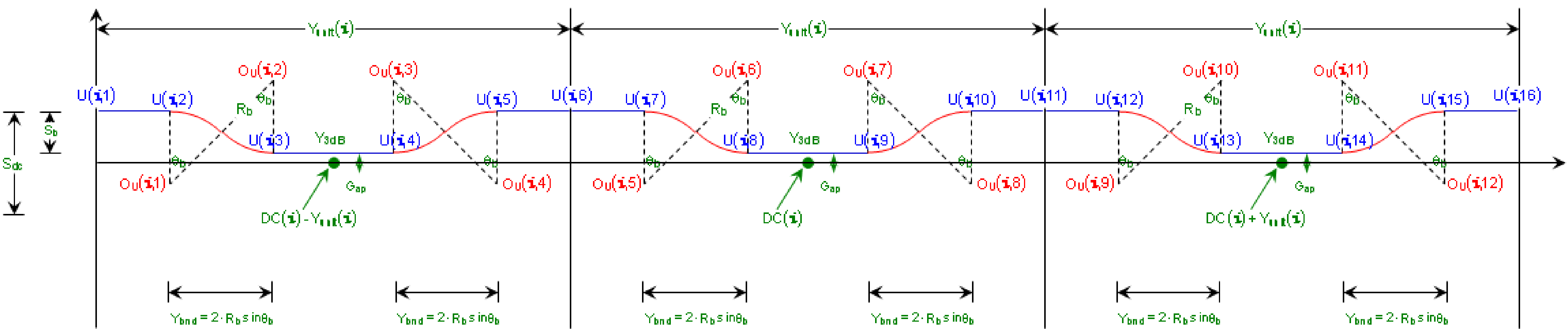
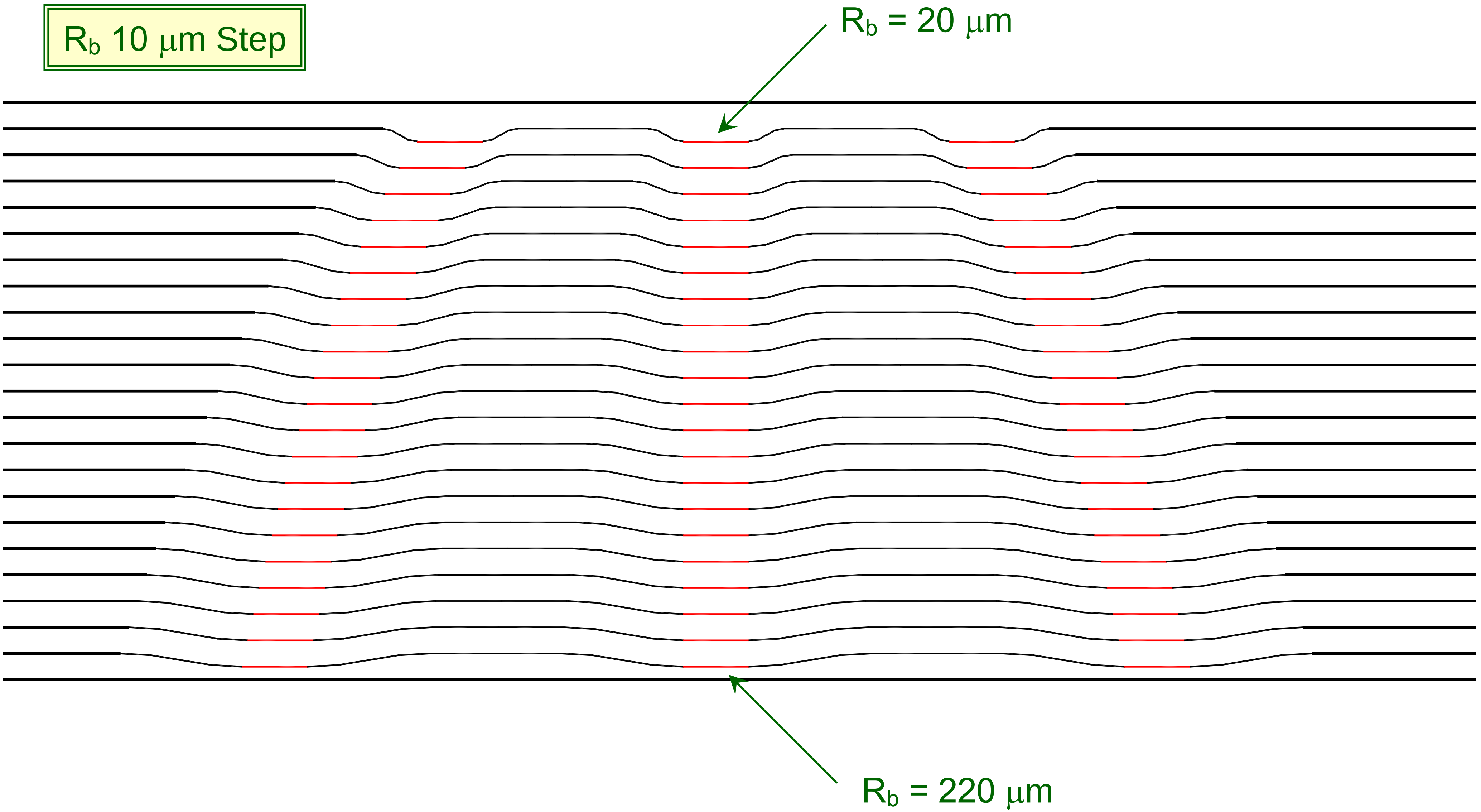


Test Radius of Curvature by S-Bends

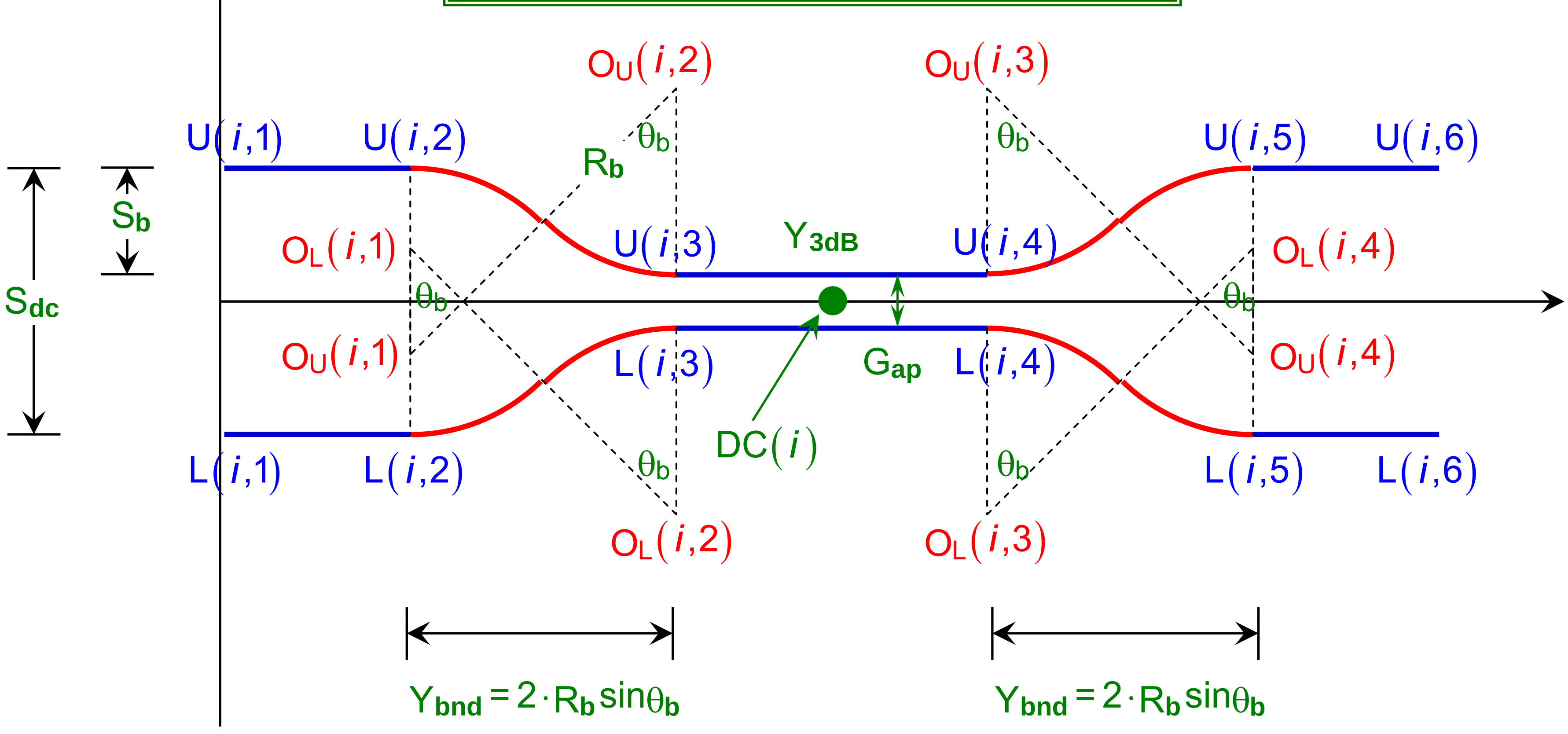


Test 3dB Directional Coupler

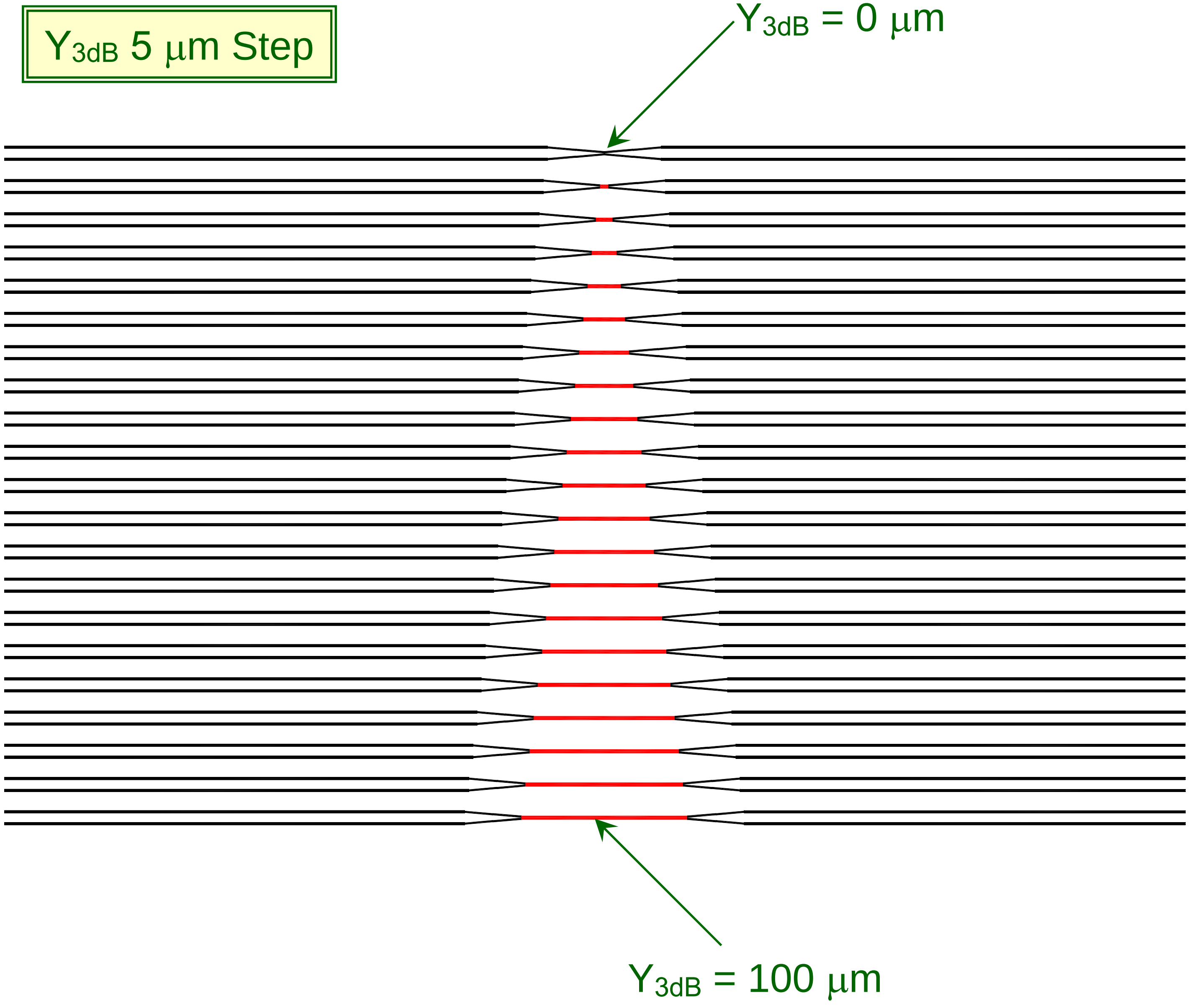
S_{dc} = 7.2 μm
G_{ap} = 0.2 μm (Core edge separation)
S_b = 3.0 μm
Y_{3dB} = 0 ~ 100 μm (5 μm Step)
R_b = 100 μm
O_{fst} = 0.03 μm

$$S_b = \frac{S_{dc}}{2} - \left(\frac{G_{ap}}{2} + a\right)$$

$$\theta_b = \cos^{-1}\left(1 - \frac{S_b}{2 \cdot R_b}\right)$$



Test 3dB Directional Coupler

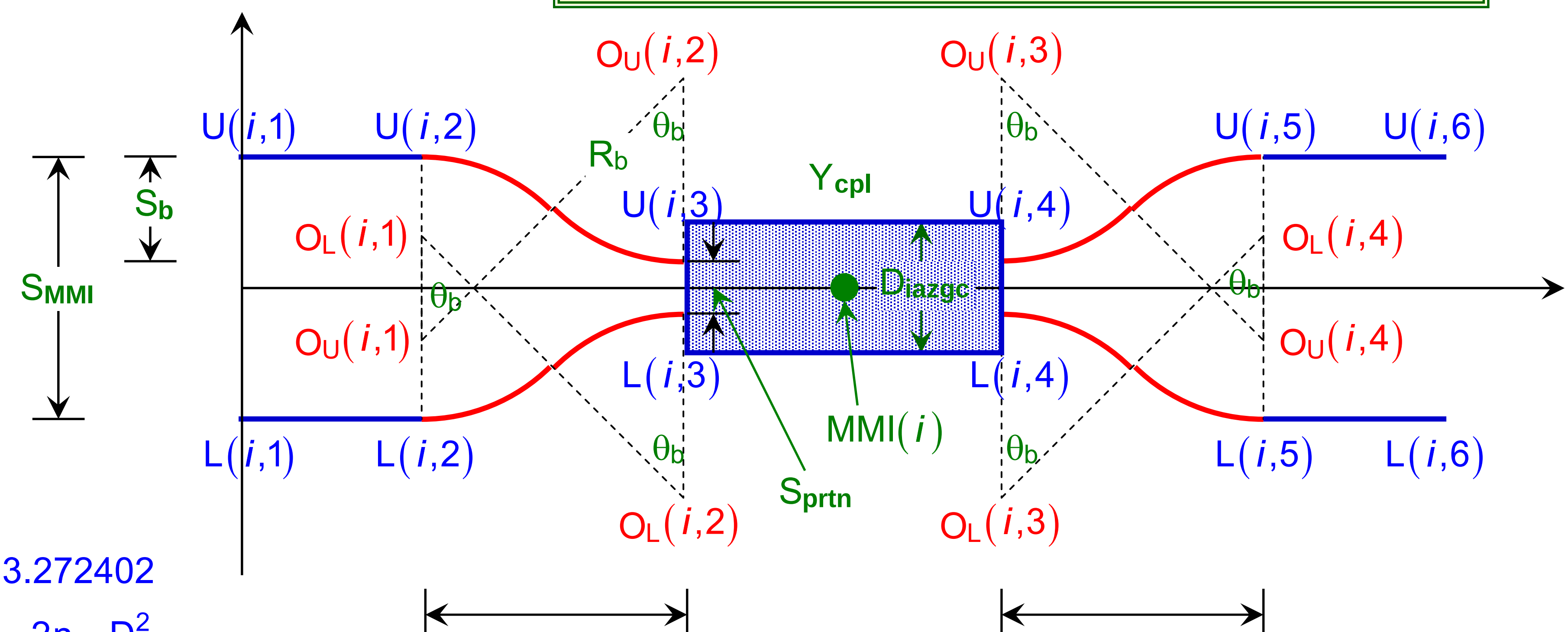


Test MMI 3dB Coupler

$$S_b = \frac{S_{MMI}}{2} - \frac{S_{prtn}}{2} + O_{fst}$$

$$\theta_b = \cos^{-1}\left(1 - \frac{S_b}{2 \cdot R_b}\right)$$

S_{MMI} = 9 μm
D_{iazgc} = 9 μm
S_{prtn} = 3 μm (Core center separation)
S_b = 3.03 μm
Y_{cpl} = 114 μm (40 ~ 240 μm : 10 μm Step)
R_b = 100 μm
O_{fst} = 0.03 μm



$$n_{eff} = 3.272402$$

$$Y_{cp\ell 0} = \frac{2 n_{eff} D_{iazgc}^2}{3\lambda} = 114\ \mu m$$

$$Y_{bnd} = 2 \cdot R_b \sin\theta_b \qquad Y_{bnd} = 2 \cdot R_b \sin\theta_b$$

Test MMI 3dB Coupler

Y_{cpl} 10 μm Step